Understanding correlations and common drivers
1. Introduction

1.1. Correlations: The business end

We often come across statements such as “the correlation between these two lines of business, CAL and PPA is, 87%”. What does this mean? Is there indeed such a thing as a correlation between two lines of business in the abstract? From the above statement one might be led into thinking that correlation is a measure of the similarity between two lines, as one might expect that the overlap of concerns between someone responsible for a portfolio of CAL and someone else responsible for one of PPA might be 87%. This may be of interest to a manager thinking to move an actuary or assessor over from PPA to CAL or vice versa. Or it might be thought that a model for PPA outstanding liabilities should resemble a model for CAL outstanding liabilities about 87%, if indeed such resemblances could be quantified or were to be of any use to anybody. All of this is vague and subjective. There is only one sense in which correlation is meaningful and relevant to a portfolio manager, one sense in which the use of the term is consistent with its origin in mathematical statistics, and that is as a measure of the synchronisation of the differences between predicted and actual loss values. The correlation that matters is the correlation in the volatility component of forecasts.

If our model for line A predicts a 12% increase in mean loss for next year over the year just ended, and our model for line B predicts a respective 14% increase, does this mean that lines A and B are correlated? The answer is no, not at all. The forecast mean losses form our best estimates, they are what we are planning on. If line A has a forecast mean loss of 110 for next year and line B a forecast mean loss of 150, and at the end of that year the actual losses for A and B are 120 and 180 respectively, is this evidence that lines A and B are correlated? Yes. It doesn't prove it, but if A and B are positively correlated this sort of result where A and B both experience a shortfall (or an overrun) in the same year is more common than not.

Here are two situations that look superficially similar. Only one of them has anything to do with correlation.

In the case illustrated on the left both lines project mean losses for the coming year as higher than the observed losses for this year. The models in each case are based on data relevant to the respective lines and they determine the reserves that will be held. Correlations do not enter into the picture. The projections are of the mean losses and these may be perfectly correct even though the observed results when they come in do not coincide with them exactly. When at the end of the year we compare the reserves based on our previous projections with the observed losses, we are faced with either a shortfall or an overrun. Correlation is the tendency for these unpredictable components to coordinate.
The existence of a positive correlation in the forecast volatility between lines means that the combined risk fund which covers for losses above the reserved amount will, on average, experience larger draws than it would if the lines were independent. Such a correlation leads to a reduction in diversification credit.

Example: Projected mean losses for Line A are 100M with a CV of 0.4, Line B mean losses are 150M with a CV of 0.3. Distributions are taken to be lognormal.

The probability of needing to draw on the risk fund is around 45% (it actually drops slightly for increasing correlations above -0.5), but given that there is such a draw the average size of it increases sharply with correlation, from 58M when correlation = 0 to 80M when correlation = 0.8.

1.2. Correlation is model dependent; there are no industry-wide correlations

The two time series in the graph below have a linear correlation of 97%.

However it is also clear that both are subject to an increasing trend. When the linear trends are removed the correlation measure falls to a statistically insignificant -1%. Are the two series correlated? The answer depends on the model. If they are each modelled as a constant plus a random factor, then the two “random factors” show a startling parallelism, justifying a correlation assumption of 97%. Clearly this misrepresents the situation, because it fails to account for the fact that each series is increasing on average. If the model is linear trend plus random factor then the variability in the observations is accounted for without the need to introduce a correlation.
How can we know that two long-tail lines are correlated? The first step is called detrending the data. This can be understood as smoothing the data down to a pattern of statistically significant trends and then replacing each data point by its difference from the respective smoothed point, the differences being known as “residuals”. This is accomplished in the Probabilistic Trend Family (PTF) modelling framework by the placement of parameters at identified change points along the development, accident or calendar axes. Once this has been accomplished the residuals for each of the two lines should appear to be randomly scattered around zero. We can then carry out a standard statistical test for correlations in the residuals.

The plots above show residuals by calendar year for two correlated segments, S1 and S2, beneath their respective calendar year trends.

In this case both segments happen to have the same trend pattern, zero from 1978 to 1986, followed by a non-zero trend from 1986 to 1984. This common pattern, however, does not represent the correlation between the segments.

The marked observations (blue line) correspond to the trace for all observations occurring in accident year 1982. The losses in calendar years 1984 and 1990 (black arrows) are lower than expected for this accident year in both lines of business. In calendar years 1982 and 1985 (red arrows), the losses are higher than expected in both lines. The trace shows that the residuals for the two segments are more likely to be both positive or both negative rather than for one to be positive and the other negative. In other words, relative to our best models for the two lines, there is a tendency for both segments to either fall short of or to exceed expectations in the same years. The volatility in the lines is positively correlated.
Understanding correlations and common drivers

Correlations are in the volatility component of a model

Two lines are (positively) correlated when their results tend to miss their target values in the same way. This is what should concern business planners, because it affects the unpredictable component of the forecasts. What is predictable, when it includes common trend patterns, as in the above example, does not count towards correlation, because its effects are already incorporated into the model and forecast. A forecast must include a volatility measure, ideally in the form of a loss distribution but at least in the form of a standard deviation.

Example of correlated residuals, $\rho = 0.598$.

The scatterplot of the residuals shows a clear relationship between the volatility of the two lines. As described previously, the two lines tend to both either have negative residuals or positive residuals. This correlation in the volatility of the losses is the critical component for the purposes of risk diversification and risk management.

1.3. Correlations are in the volatility component of a model

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1.4 Common accident year and common calendar year drivers

Common drivers are a stronger influence than correlation. However, they are not typically found outside closely related losses. For example, Gross versus Net of Reinsurance (Net of Reinsurance is a subset of Gross so common drivers are expected), layers (layers are subsets of ground up losses), and segments of the same line. In this respect, detection of common drivers is as important as understanding correlations. The two effects must be correctly distinguished and adjusted for as management strategies of these risk components differ.
1.5. Case studies

The remainder of this document consists of a series of case studies illustrating the various types of correlation, common drivers, and the impact on reserve risk, underwriting risk, and the combination of reserve and underwriting risk.

Section 2: Common calendar year drivers
In this section, three examples of common calendar year drivers are considered. The first example describes Gross data and Net of Reinsurance data for an E&O D&O line. The trend structure is almost the same, especially along calendar years. That is, the trend changes occur in the same periods, indicating potential common drivers. The resulting volatility (process) correlation and parameter correlation are high. The second example illustrates the common trend structure identified in layered data - again, especially calendar year trends. Volatility correlations and parameter correlations are high. Finally, the last study in this section demonstrates common calendar year drivers for two LOBs (same line, different states) where volatility correlation is also statistically significant but not high.

Section 3: Common accident year drivers
The case study considered in this section consists of two segments of Worker’s Compensation. In the identified Multiple Probabilistic Trend Family (MPTF) model for the two segments, accident year levels are fitted to each segment and changes occur synchronously. The synchronous changes indicate common accident year drivers. The process correlation remaining after these accident year level changes are accounted for is relatively low. The common accident year drivers form a stronger relationship than correlation. They have major impact on pricing future accident years (Section 4).

Section 4: Common accident year drivers and pricing future accident years
The two segments of Worker’s Compensation from Section 3 are revisited for the purpose of calculating the future ultimate loss distributions. The importance of the common accident year drivers are discussed in relation to the identification of appropriate future forecast scenarios.

Section 5: Spurious correlation
This section discusses spurious correlation as a result of an incorrect model for the data. To illustrate this point, two LOBs are simulated independently each with their own unique trend structure. The only material difference in the LOBs is that one LOB has a calendar year trend of 10%, the other of 20%. In a correct model of the underlying data process, the model would recognise that each LOB has a separate trend for each direction and a process correlation of zero - since this is how the data are generated. If an incorrect model is used, one that does not describe the calendar year trends, then measuring the correlation between the two sets of residuals is meaningless since the residuals are not random from one distribution (ie, iid). This measurement of correlation is spurious since it arises as a result of calendar year trends being present in both LOBs which are not included in either model. The correlation is an artifact of a model which does not fit all the trends in the data.

Section 6: Companies versus the Industry
Two examples are presented in this section. The first example considers an Auto BI study of Company A versus the Industry. The trends identified are very different. Company A has stable calendar year trends whereas the industry’s calendar year trends are unstable. The Industry has a strong development trend decay whereas Company A’s data demonstrates no measurable trend decay. Volatility correlation between the company and the Industry is low at 24%.

The second example considers data extracted from A.M. Best schedule P for two selected writers of PPA and CAL. The trends identified for each company are compared with the Industry along with the volatility correlations between PPA and CAL - in each company and the industry. We demonstrate that once all trends in the three directions (development, accident, and calendar), volatility correlations between PPA and CAL are not detected - for either company or the industry.
Section 7: Risk capital allocation
In this case study, risk capital allocation is considered for six LOBs. The allocation is based on the variance-covariance formula. Risk capital can be allocated across LOBs or by calendar years. Note that the trend structure of each LOB is distinct and only two LOBs (LOB 1 and LOB 3) have significant, but relatively low, process (volatility) correlations.

Section 8: Reserve, underwriting, and combined risk
This case study measures the reserve, underwriting, and combined risks for the six LOBs considered in section 7. Reserve risk and pricing risk are not two unconnected problems; a high proportion of policies in the next underwriting year are renewal insurance. Writing another (next) year, is likely to add to the risk diversification. Any significant correlations are driven by common parameters - another reason to consider reserve risk and underwriting risk jointly.

Section 9: Common accident year drivers and the reserving cycle
The so-called reserving cycle is a common feature of company’s booked reserves as company’s respond to market pressure. The reserving cycle is driven by the premium cycle not by the best estimates of ultimates.

In this case study, we consider the top ten writers of Commercial MultiPeril in A.M.Best 2011 based on reserves held at as at year end 2011. The behaviour of the booked ultimates at the beginning of each policy year is compared to the independent estimate of the ultimates as at year end 2011. The loss ratios are shown to be deteriorating since 2008 in the industry and (in general) for each of the ten companies considered. Further, the booked ultimates at the start of the policy year versus the independent estimates of the ultimates at year end 2011, show a clear indication of the reserving cycle in action. That is, the independent estimates of the ultimates indicate that the booked reserves as at year end 2011 are optimistic for the most recent accident years (costs are increasing and company’s need to appear profitable and the ultimates are likely to be revised upward - based on the independent result), whereas in the early accident years (2002--2006), the ultimates estimated are reduced from their initial estimate.

This is completely in line with our expectations of the market process, but this behaviour should not be seen as indicating correlation between the losses within the lines. This ‘correlation’ is a feature of the common market pressure on estimates - not on the true estimates of the liabilities for each company - and is identified as spurious.
2. Case study: Common calendar year drivers

A series of examples are provided to illustrate the concept of common calendar year drivers and implications in managing risk. The following categories are considered:

- Gross versus Net of Reinsurance (E&O and D&O)
- Layers
- Distinct lines written by the same company

Although not comprehensive, the above list serves as a solid basis to the concept of discussing volatility risk, common calendar year driver risk, and understanding the difference between volatility risk (as a result of process correlations) and common calendar year driver risk.

2.1. Gross versus Net of Reinsurance

In this first example of detecting and quantifying common calendar year drivers, Gross versus Net of Reinsurance for E&O ad D&O data, common calendar year drivers are expected to be found. Net of Reinsurance is a subset of Gross and therefore common features are to be expected, but are not always found. Trends, especially calendar and accident, are closely related. The comparable models for Gross (left) and Net of Reinsurance (right) are shown below.

The model trends are very similar; trend and volatility changes usually coincide. The critical trends in common are the calendar year trends (below) and accident year level changes. Common calendar year drivers are clearly visible as the trend changes occur at the same point.
Similarly, the process volatility is closely related. A scatter plot of the residuals, from the respective Gross and Net of Reinsurance models, exhibits a clear (linear) relationship; a correlation of 0.854.

The residuals by calendar year are illustrated below where contouring indicates the general movement in the residuals which can be seen to be moving synchronously. A trace (following page) serves to illustrate this.
For the model described above, the residuals by accident year traced for the last calendar year are clearly correlated; when a value in a year is low/high in one segment it is usually low/high in the other segment also at the same time.

The residuals from both lines of business are statistically indistinguishable from two normal distributions. Thus, the process correlation can be considered the volatility correlation between two normal distributions.

If only an average calendar year trend is fitted to each segment, the result is the display below which highlights the combination of the synchronous trend changes and the volatility correlation.

The accident year 1987 is traced in the residual displays; the correspondence in the trends and volatility are clearly visible.

This case study illustrates the worst possible relationship between two Lines of Business (if in fact they were separate lines); namely common drivers (accident year and calendar year) and volatility correlation. In almost all cases, this proximity of relationship is only expected when the LOBs analysed are in fact subsets of one another.
2.2. Layers: Limited to 1M, 1Mxs1M, and Limited to 2M

In this next example, data are split into three layers - paid losses with each individual loss limited to 1M, paid losses with individual losses in excess of 1M with the excess limited to 1M (1Mxs1M), and paid losses limited to 2M. Similar trend structure and common drivers are expected since 1M + 1Mxs1M = 2M.

The Layer 1M has a higher inflation rate than 2M, and 1Mxs1M has inflation rate that is insignificant. If the only available array is 1Mxs1M then it would be prudent not to set the inflation to zero, as process volatility is high. One could argue that positive inflation is present, and we have a very uncertain estimate of it (5.63%+/-4.11%). If any one of the other two arrays is available the very high process (volatility) correlation between the layers reduces parameter uncertainty in the composite model. In this case there is convincing evidence that inflation for 1Mxs1M is zero.

The three residual displays by calendar year for the layers exhibit very high process correlation.
Understanding correlations and common drivers

When the composite model is optimized some trends in the data are found to be common (red bars / lines indicate common parameters) between the layers and for 1Mxs1M the calendar year trend is zero.

Having recognised the same trend structure (and common drivers) in the three layers, the efficiency of the reinsurance program (in terms of reducing risk capital as a proportion of the mean reserve) can be assessed.

Indeed, the CV of the aggregate reserves for 1M and 2M are the same (0.15). That means that both reinsurance (ceding) programs are equally efficient!
2.3. LOB 1 and LOB 3

The optimal model for the two LOBs, LOB 1 and LOB 3, is shown below.

As with the layers, this model shows common calendar year drivers affecting both LOBs. Where trends in two different lines are exactly the same (indicated by the red bar in the model displays), the common trends have a correlation of 1. If the two estimates of the calendar year trends were independent, common calendar year drivers would be suspected as the changes in calendar year trend occur at the same time. Synchronous changes in trend are a key indicator of common drivers.

Furthermore, the LOBs not only have common drivers, but the process volatility between the LOBs is also correlated as illustrated below.

There is a reduction in risk diversification credit from writing these two lines by way of the common parameters and process correlation. The reserve correlations (0.821) are much higher than the process correlation. This unusual case is a result of the most recent calendar year trends for the two segments LOB 1 and LOB 3 being set to be the same for each line in the future.
To see the effect of the inclusion of the correlation and common drivers on the risk capital, the risk capital is calculated for two models: a model were the correlation is explicitly set to zero and no common trends are applied - that is, only the data in each LOB is used to estimate the trends (development and calendar) - and a model where the common trends and process volatility is included. The risk capital requirement assuming independence is then compared with the risk capital calculation where the process correlation and common drivers are applied.

For comparison, the value-at-risk at the 95th quantile (percentile) is calculated for both models: independent (left) and incorporating the common drivers and process correlation (right).

Similarly, the Solvency II one year risk horizon metrics (see Solvency II - One-year and ultimate year horizons for long tail liabilities) are calculated for the model for the two lines assuming independence (left) and the optimal model with both common calendar year, development year trends, and process correlation (right).

The required technical provision has increased from 1.693B (assuming independence: left) to 1.732B (common parameters and process correlation: right). The additional 39M (approximately 2.3%) is the penalty for the lack of risk diversification.
3. **Case study: Common accident year drivers**

In this case study, common accident year drivers are demonstrated in the context of two segments of Worker’s Compensation: SAD and SAM.

The two segments have changes in accident level in common and also demonstrate synchronous changes in level. The synchronous changes in parameters are a critical component of identifying common drivers whether by accident or calendar year.

3.1. **Worker’s Compensation Segments: WC SAD and WC SAM**

Consider the following two segments of Worker’s Compensation written in California: SAD (left) and SAM (right). The red bars indicate common parameters between the segments. Although the calendar and development year parameters vary slightly, the accident year parameters move synchronously thus making the mean ultimates vary synchronously (but this is not correlation).

Both sets of residuals can be assumed to originate from a normal distribution (and are randomly distributed), so the process correlation (0.249) below is the volatility correlation between two normal distributions.
Understanding correlations and common drivers

If the common accident year movements are ignored and the average accident year level fitted to both segments, then a very high spurious correlation measure of 0.897 is obtained.

Above, the residual displays with scatterplot for SAD and SAM are shown for a model which does not describe the accident year changes. The spurious correlation (0.897) is very high, but it should be immediately apparent that this is not all correlation in randomness - there are distinct changes in level across the accident years (as indicated by the red arrows).
Understanding correlations and common drivers

The high correlation is an artifact of a model which does not fit all the trends in the data. Instead, the correlation reflects the commonality of the trends rather than process (volatility) correlation.

However, the correlation is no longer the correlation measured between two (normal) distributions; the means vary over time in both sets of residuals. For instance, residuals for accident years 90-93 have a positive mean, whereas 87-88 have a negative mean.

In the correct model (page 12), with the accident year levels correctly fitted, the correlation between the segments is predominantly the volatility correlation. The result of the fitted accident year levels is that the mean ultimate losses (by accident year) move synchronously (common drivers), however the risk factors arising from volatility are not as severe (volatility correlation is only 0.25). The accident year levels moving together is a much stronger relationship than volatility correlation.
Understanding correlations and common drivers

The mean ultimates move synchronously (left) and a graph of the mean ultimates of SAM versus the mean ultimates of SAD (right) shows an almost perfect linear relationship.

The reserve distribution correlation is only 0.086! The reserve correlation is the correlation in the losses not explained by the means – and therefore is the critical measure when evaluating risk diversification. Models that do not capture the trends in the three directions in the data may indicate spurious correlations and erroneous conclusions. It is also important that the weighted standardised residuals of each model can be regarded as a random sample from a (normal) distribution. This way, the process (volatility) correlation can be measured correctly.

These features are also illustrated in the accident year summaries for each segment displayed below.
Understanding correlations and common drivers

Case Study: Common accident year drivers and pricing future accident years

In this example we continue with the two segments of Worker’s Compensation SAD and SAM. These two lines demonstrate common accident year drivers.

The impact of common accident year drivers must be considered when pricing future accident (or underwriting) years. The close relationship of the accident year parameters are considered in respect of future forecast assumptions.

The linear relationship in mean ultimates is important when forecasting future underwriting (accident) years. If the accident year level for one segment is expected to increase by 10% +/- 2%, then the other segment is also likely to increase by 10% +/- 2% in the same accident year. The relationship in the mean parameter estimates is not volatility (risk) correlation and does not indicate lack of diversification. The movement in means may be able to be related to internal or external drivers - and risk exposure can be managed. Correlation in risk is significantly harder to manage as it invokes correlation in the random component - variation which is not readily able to be connected to any internal or external drivers.

The synchronous movement in the accident year trends is readily observed in the model displays shown previously (page 16). The correlation between the mean accident year level parameters, provides an idea of the closeness of the relationship. This correlation, between the two sets of mean accident year level parameters, is: 0.995. This measure provides support that if a level change is expected to occur in one segment, then a corresponding level change is expected to occur in the other. This correlation measure is not able to be identified prior to analysis (identification of trends), nor does it necessarily imply the magnitude of the change in parameter levels are the same (although in this example the changes in mean level are essentially identical as a result of the constraints between the segments - it is a feature of the model). It does, however, emphasize the importance of adjusting accident year levels for both segments simultaneously.
The relationship in the mean parameter estimates is not volatility (risk) correlation and does not indicate lack of diversification. The movement in means may be able to be related to internal or external drivers - and risk exposure can be managed. Correlation in risk is significantly harder to manage as it invokes correlation in the random component - variation which is not readily able to be connected to any internal or external drivers.

The close relationship between the two segments does not eliminate the risk diversification credit for combining the analysis of the reserve distribution with the future accident (underwriting) year (see Modelling multiple lines of business brochure and Pricing: Segments, Layers, and Reinsurance brochure). The joint increase in parameters (with the associated uncertainty), increases the over-all risk diversification as the uncertainty in the parameter estimates is not highly correlated between the segments.
5. **Case study: Spurious correlation - an artifact of the wrong model**

In this section, two case studies serve to demonstrate spurious correlation measurements. In both cases, deficiencies in the model lead to incorrect correlation measurements. When all the trends in the underlying data are correctly measured and adjusted for in the models, then no volatility correlation is identified. This reaffirms that correlations are model dependent.

The first case study considers two LOBs which are simulated independently. Both LOBs have a single development year, accident year, and calendar year trend. In a correct model of the underlying data process, the two LOBs would have a trend for each direction and a process correlation of zero - since this is how the data are generated. If the calendar year trend is not described by the model, then spurious high correlation of 0.699 is found. Only the approach which matches the data generation is important - the differences in projections is substantial.

5.1. **One trend for each direction: Ex 3, Ex 4**

The data SDF Ex3 Ex4 consists of two datasets simulated independently. The data Ex3 is simulated with a 10%+ _ calendar year trend whereas the data Ex4 is simulated with a 20%+ _ calendar year trend. Otherwise, the statistical features of the two simulated datasets are identical. The two models correctly fitted to the data are shown below - the parameters estimated from the data are very close to the true parameters (as we would expect for simulated data).

In the event that the trends are described accurately (as above), the correlation between the two segments is expected to be insignificant and close to zero. If we estimate this correlation, this is exactly what we find.

The process (volatility) correlation measured from the residuals is 0.098 which is found to be statistically insignificant (thus the entry in blue). In contrast, consider if the respective calendar year trends are not fitted to the data. The calendar year residual displays are shown below to emphasize the correlation metrics (first accident year marked).
Understanding correlations and common drivers

The residuals are clearly highly correlated (but they do not come from the same distributions across the calendar years). The residual is clearly a function of time in that early calendar years are highly likely to contain negative residuals (both LOBs) and more recent calendar years (post 1988) positive residuals. If we measure this correlation we find it to be 0.699.

The correlation is very high (0.699) and is statistically significant. This result is purely a reflection of calendar trends being present in both datasets which are not described by either model. The correlation is spurious and simply measures trend structure which is not captured.

5.2. Spurious correlation between Industry PPA and CAL data

As was shown in the previous case study, spurious correlation is introduced by failing to detrend the data in the three directions. The correlation measured was spurious as there were trends in the data not described in the models. Once these trends were accounted for, the correlation was statistically insignificant.

In this second example Paid Losses for the Industry PPA and CAL data from AM Best (2011) are modelled using the Mack method. The residuals are shown by Calendar year for CAL (left) and PPA (right) below with the trace line for accident year 2004 highlighted.
The marked residuals for the Mack method exhibit correlation (by eye). This correlation is then measured and shown in the residual scatter plot of PPA vs CAL below.

Although the correlation is strong in the residuals, like the example previously, this correlation is also spurious. The calendar year residuals show the Mack method is over fitting the data more recently - a clear negative trend is evident in both residual displays (though it is much stronger for PPA). The volume weighted average link ratios (of which the Mack method is the regression formulation) do not describe the salient features of the data and, as a result, there is correlation found between the lines which does not exist in a correct model for the data (see Section 6.2).

The method has not described the trends in the data in either the calendar year direction or the accident year direction - see the full residual displays against each trend direction below.

The Mack method (and in fact all link ratio methods - including bootstrapping from link ratio models) are inappropriate for both these LOBs. Link ratio methods cannot describe changing calendar year trends yet, as seen in Section 6.2, changing calendar year trends are found in this data.
Case study: Companies versus the Industry

The application of measuring correlations in the industry and using these figures as benchmarks or figures for companies to use is analogous to calculating correlations between the performance of different classes of vehicles and applying these correlations to all manufacturers and expecting the manufacturers to be able to use the correlations in some way. While these figures may be interesting in aggregate, they are of no relevance to the individual manufacturers.

What is true for aggregate data is not necessarily true for pieces comprising the aggregate – in fact, opposite conclusions can be drawn (see Simpson’s Paradox). Simpson’s paradox typically arises from combining data where unequal group sizes in the presence of lurking variables (typically inflation in our case) results in incorrect conclusions being drawn. The easiest way to avoid such anomalies is to examine each company, including the relationships between lines written, on its own.

Individual companies do not exhibit the same trend structure as the industry. Two examples are considered and for each example we find different trends in the companies versus the trends found in the industry.

6.1. Company A versus the Industry

The following discussion relates to Auto BI written by Company A (representing about 3% of the industry) and the Auto BI industry data: MAA951. The industry data have high unstable calendar year trends and a final negative development year trend, whereas Company A, on its own, has an insignificant (zero) calendar year trend and an insignificant (zero) final development period trend.

The optimal model for the company is on the left; the optimal model for the industry on the right. Note the difference in both development and calendar period trends. The industry has clear, volatile periods of inflation whereas there is no evidence of this in Company A. Furthermore, the industry data shows a definite decrease in payments after development period 14 whereas there is no evidence (yet) of decreasing levels of payments in Company A.
Understanding correlations and common drivers

Process correlation between the company and the industry data is statistically significant but low at 0.240. The low process correlation between the company and the industry does not lend credibility to using correlations obtained between industry LOBs to represent correlations within Company A’s LOBs.

The industry data can be used to credibility adjust the trends in the model for Company A. For instance, the calendar year trend in Company A can be optimised to be the same as the base trend in the industry where it should be noted the industry calendar year trend is no longer zero either.

Some parameters in the development direction are also found to be the same, but the trends in the company’s data are not the same as in the industry. The zero development period trend (in the tail) for the company is not credibility adjusted to the industry as there is no statistical evidence to do so. Given the low process correlation between the company and the industry it is unlikely that the same process correlation effects measured from the industry between multiple lines would apply to Company A’s equivalent lines.
6.2. Companies LMI and TG, versus the Industry for CAL, PPA

A.M. Best Schedule P data (2011) are used to compare CAL and PPA for two companies, LMI and TG, with each other and the industry. It has been mentioned several times in this brochure that the industry may expect CAL and PPA to be highly correlated. However, what do the data say?

As mentioned at the start of this brochure, correlation only has meaning relative to mean predictions (that is, correlation in volatility). If we calculate spurious correlation between the paid losses in the two LOBs (PPA and CAL) for the Industry, Company LMI, and Company TG, then we obtain the following matrix.

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<td>0.864</td>
<td>0.958</td>
<td></td>
<td>0.854</td>
<td>1.000</td>
</tr>
</tbody>
</table>

The paid loss data, with no adjustment for trends, is showing the high spurious correlation of about 0.87 between PPA and CAL in the Industry, with slight variation in the two companies. However, when the trends in each LOB have been fully adjusted for, the resulting correlation matrix is very different.

<table>
<thead>
<tr>
<th></th>
<th>PPA</th>
<th>CAL</th>
<th></th>
<th>PPA</th>
<th>CAL</th>
<th></th>
<th>PPA</th>
<th>CAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>1.000</td>
<td>0.000</td>
<td></td>
<td>0.477</td>
<td>0.000</td>
<td></td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>0.000</td>
<td>1.000</td>
<td></td>
<td>0.000</td>
<td>0.304</td>
<td></td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>TG</td>
<td>0.477</td>
<td>0.000</td>
<td></td>
<td>1.000</td>
<td>0.000</td>
<td></td>
<td>0.000</td>
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<td></td>
<td>1.000</td>
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</tr>
<tr>
<td>LMI</td>
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<td></td>
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<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
<td></td>
<td>0.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

All the correlations between PPA and CAL are statistically insignificant (thus grey zeroes), and only correlation between company TG’s LOBs and the respective Industry LOBs remain. Given the small sample size in Schedule P it is hard to interpret the meaning of the correlation (Company TG comprises roughly 5% of the industry for these lines), but it seems that the risks taken by company TG are similar to the risks encompassed in the whole industry.

Note that reserve distribution correlations are typically much lower than volatility correlations.
Calendar year trends are shown below.

The industry calendar year trends are similar for both PPA and CAL; they are both less than average economic inflation over this period. The two companies show different calendar year trends from each other and the industry for both lines of business. TG PPA is close in structure to the industry PPA. Given calendar year trends include both economic and social inflation, it is unreasonable to expect individual companies and the industry to have the same calendar year trends.
Understanding correlations and common drivers

Case study: Risk capital allocation

Reserve correlation is an important component of the allocation of risk capital and as a measure of risk diversification (reserve correlation between LOBs). Reserve correlations within LOBs are primarily a function of parameter uncertainty; the higher the parameter uncertainty, the higher the correlation between cells.

As the probabilistic trend family modelling frameworks comprise a clear structure for relating cells by trends with associated uncertainty, it is no surprise that the correlations between cells and across LOBs are included in a natural way within this framework.

Risk capital allocation accordingly to variability can be calculated directly using the variance-covariance formula. This formula can be used to allocate capital across LOBs, across calendar/accident periods, or both.

Risk capital allocation across six LOBs using the variance-covariance formula

Risk capital can be allocated between LOBs and across calendar years by the same variance-covariance formula.

Percentage allocation to the $i$th line, $L_i$, is:

$$A_i = \frac{\sum_j C_{ij}}{\sum_{ij} C_{ij}}$$

Where $C_{ij}$ is the covariance of $L_i$ and $L_j$.

The formula can be extended to include time (either calendar or accident period), by summation of the covariances across the relevant time period. Similarly, allocation across time periods for a single LOB can be readily considered by treating $i,j$ as time indices rather than LOB indices – the formula still holds.

The above forecast summary is for six LOBs. Clearly, LOB 4 is expected to take the most risk capital followed by LOB 3 – just based on the relative standard deviations.
Risk capital allocated across LOBs can be calculated in general (forecast summary) or using specified value-at-risk (V@R) levels using the Predictive Aggregate Loss Distributions (PALD) simulations. Since there are no analytical distributions for the aggregate of lognormals, simulations from the correlated lognormals in all the cells are produced in the PALD module to obtain distributions of reserves by accident year, calendar year, and the total. These simulations can then be used to calculate percentiles, V@R, or other statistics.

Similarly, the allocation by accident period and calendar period for the aggregate of the six LOBs is as shown above. Similarly breakdowns within each LOB can also be calculated (not shown) where the allocation within an LOB follows the risk characteristics of that LOB.
8. Case study: Reserve, underwriting, and combined risk

A single composite model measures the reserve, underwriting and combined risks for each LOB and the aggregate. Reserve risk and underwriting risk are not treated as two separate analyses; rather, the same model can be applied for either, along with analysis of the combined risk.

Combined risk is less than the sum of reserve risk and underwriting risk.

This is an important result typically ignored when considering reserving versus underwriting risk even when the majority of business underwritten in the next underwriting period is renewal business. The mix of risks in the underwriting period is usually the same as the reserving period.

Reserve distribution correlation is usually very low between reserve and underwriting risk. Any correlations between future and reserve periods are driven by common parameters. That these parameters are common is another reason not to separate the reserve and underwriting calculations.

The forecast table excerpt above corresponds to the six LOBs presented previously but where one future underwriting period has been added to the forecast scenario. The reserve and underwriting distributions are forecast jointly to calculate the total reserve for the combined reserve and underwriting periods. In this way, risk diversification by writing multiple underwriting periods is correctly included in the analysis. Furthermore, it is incorrect to assume that high reserve distribution correlation exists between reserve and underwriting periods in that a higher than the mean loss in the reserve periods necessarily results in a higher than the mean loss in the future period. This result does not negate the presence of the reserve and underwriting cycle, however, since these cycles are independent of the data and are rather imposed by the nature of the reserving and underwriting methodology.
The risk capital allocation table for the reserve, underwriting, and combined estimates of reserve mean and risk capital (V@R at 95%) is shown below. While the means are additive, the risk capital clearly is not. Furthermore, if the reserve and future underwriting periods were highly correlated then risk capital for the combined forecast would be close to the sum of the risk capital for the individual pieces.

In this example, a 12% discount in risk capital is obtainable as a result of risk diversification credit between both writing the multiple lines and by combining reserve and underwriting risk. Note that for LOB 4 – the line with the greatest risk capital requirement, minimal diversification credit is obtained for combining reserve with underwriting risk.

The above display highlights the risk diversification credit gained by recognising the nature of the reserving and underwriting problem. The inner pie chart shows the risk capital allocation percentages should reserve and future underwriting risk be considered independent. The outer ring shows the assessment of the combined risk along with the diversification credit (12%) arising from the reduced risk capital requirement when considering the diversification between reserve risk and future risk.
Understanding correlations and common drivers

9. 

**Case study: Common accident year drivers and the reserving cycle**

This case study considers the evidence of a reserving cycle and demonstrates that this cycle is not a feature of the long-tail liabilities but rather is a result of market pressure and methodology. The rationale behind the reserving cycle is described. The A.M. Best (2011) Commercial Multi-Peril data are then examined for evidence of this cycle. While the booked reserves do provide some evidence of a reserving cycle, the long-tail liabilities do not show any evidence of common accident year drivers. The apparent correlation in the behaviour of the booked reserves is methodological and not a feature of the data.

A common fallacy in the industry is a belief that long-tail liability losses exhibit a reserving cycle. The actual losses from long-tail liabilities do not follow a reserving cycle. Booked reserves and premiums, however, may.

Premiums are set based on the demand for retaining market share and competitiveness amongst economic conditions. If the market underprices the risk, then individual companies will also underprice risk in order to maintain market presence. As a result of common commercial interests, there is a definite element of herd mentality.

Booked reserves follow this cycle as management are biased to select the lower actuarial reserve estimates in times of underpricing risk to maintain apparent profitability. Similarly, booked reserves and prices rise as the market responds to catastrophes (and management is under pressure to be conservative).

If this booked reserve estimate pressure was not bad enough, actuaries that use Bornhuetter-Ferguson methodology are even more at risk as this method introduces spurious correlation between premiums and booked reserves before further management bias.

The cycle is this:

- Competition is low due to insurers leaving the market due to catastrophes (whether on the loss or asset side), prices rise, booked reserves are high.
- As prices rise, profits increase, more players enter the market.
- More players result in competition resulting in decreasing prices, lower reserves are booked.
- A catastrophe occurs resulting in players leaving the market
- And the cycle starts again...

True best estimates of long-tail liabilities do not respect the market’s business cycle but rather reflect the true risk of the business written. Typically, most companies write the same mix of risk from year to year. The prudent management team realises this and both sets prices and reserves according to the level of risk taken.

9.1. **Illustrative example: A.M. Best Schedule P Commercial Multi-Peril (CMP)**

In 2011, ten company groups wrote over 50% of the total reserves of US Commercial Multi-Peril (CMP) based on reserves held (where reserves held are defined as the sum of Case Reserve Estimates and Bulk & IBNR).

Although the loss ratios (for Ultimate Earned Premium) for the Industry are still healthy, it is is clear that conditions are worsening more recently. This may be a result of the GFC impacting businesses making claims on CMP insurance coupled with reductions in market share (as a result of businesses folding, reduced value, etc) and other recent catastrophes.
Understanding correlations and common drivers

The position in the cycle is probably between the competition for market presence amongst difficult conditions and a catastrophe occurring. Conditions have worsened, loss costs are increasing while total premiums are decreasing (see above right), but the industry (as a whole) is still profitable (after allowing for ALAE). What is happening to booked reserves versus premium? Are the company’s pricing their risk accurately?

Industry mean ultimate loss ratios booked reserves versus independent estimates

The mean ultimate loss ratios for ultimates held in the Industry are compared with the mean ultimates estimated from the Probabilistic Trend Family (PTF) modelling framework for an optimal model and future forecast scenario. The key element here is that Insureware’s estimates are based only on trends and volatility found in the Industry data and future expectations are thus independent of both market pressure and other commercial considerations.

What we expect to see is that as the conditions worsen from 2007 onward, the mean loss ratios do not increase as greatly for the booked reserves versus the Insureware estimates of the mean ultimate loss ratios. That is, we expect the Industry to be more optimistic about the mean ultimate loss ratios due to collective decreasing of booked reserves in connection with the lower premium raised.

The above graph illustrates that the industry is behaving exactly as expected. Insureware’s estimates of mean ultimate loss ratios are not biased by management or other external commercial pressures and are more optimistic during the good years (2006—2007) and significantly more pessimistic during more difficult market conditions. This collective response to changing market conditions further reinforces the belief that the risk in the industry and individual companies are highly correlated.

The reserving cycle

In order to determine the effect of the reserving cycle, we compare the estimated ultimate at the start of the policy period versus the projected ultimate as at year end 2011. If the cycle exists then this will be illustrated in the difference between the two ultimates responding to market conditions.
Understanding correlations and common drivers

The early accident years (up to 2006) are consistently conservative. That is, for the largest ten writers of CMP, the ultimates are estimated very conservatively with the result that by year end 2011, the estimates of the ultimates have been revised downward. For 2007–2009, the ultimates are still being estimated conservatively (relative to the independent mean ultimate as at year 2011), but with decreasing conservatism. In the most recent two accident years, the company ultimates are considerably more optimistic – reflecting the effect of the reserving cycle with the tighter market.

**Probabilistic Trend Family models for the largest ten writers of CMP do not demonstrate common accident year drivers**

Below are the model displays for the ten largest writers of CMP by reserves held. The trends in the three directions and volatility are displayed (left to right: development year trends, accident year trends, calendar year trends, volatility by development period). The key components of note are that: a) the trends in the three directions are unique to each company, and b) there are no indications of common accident year drivers. The latter is expected should the reserving cycle be a feature of the long-tail liabilities.

The lack of common accident year level changes between the top ten writers of CMP (despite loss ratios behaving similarly), emphasizes the conclusion that the reserving cycle is not a feature of the long-tail liability losses but rather reflects management’s selection of booked reserves from the range of actuarial estimates. This common management bias is also not correlation.