Solvency II - one-year and ultimate year risk horizons for long-tail liabilities

- Economic Balance Sheet
- Solvency Capital Requirement
- Technical Provisions
- Market Value Margins
- IFRS 4 Phase II
- Fungibility, and Ring Fencing
- Consistency of Metrics on Updating

Insureware
Software Solutions and eConsulting for P&C Insurance
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One Year and Ultimate Year Risk Horizons

Solvency II has been designed to establish a consistently improved level of policyholder protection by means of a three pillared process which combines the quantifying and management of risk, supervision and oversight of risk policy and transparent reporting standards. New challenges to actuaries arise in the first pillar; in particular in developing accurate risk measures based on stress scenarios. In recent years actuaries have added the Mack method and the bootstrapping technique to their toolkit in order to produce distributions which can be associated with their point estimates. Whatever the value of these methods for other purposes, they are grossly inadequate for Solvency II since they do not directly model the process volatility or identify the trends in the data; they certainly cannot take a given stress level as input and produce the correctly modified loss distribution as output.

Solvency II for long-tail liabilities requires precise calibration of Solvency Capital Funds to a mandated stress level over the one-year horizon. Calibration must be done for individual Lines of Business (LOB) as well as aggregates of multiple Lines of Business, and possibly under required ring-fencing rules. Actuaries will need access to accurate and precise distributional information about future cash flows and their modifications under stress. Only a unified approach to reserving which treats trends, volatility and correlations under a single distributional paradigm can achieve this result. In this article we discuss the probability distributions and correlations needed for the calculation of Solvency II risk measures for long-tail liabilities to be included in the economic balance sheet. Distributions of the payment streams by calendar year, conditional on the first calendar year being in distress, are critical to the calculations.

We argue strongly that an integrated approach that accurately measures the trends and volatility in each LOB, and the different types of correlation between them, is required to obtain precise Solvency II risk measures that are also consistent on updating. We present a tractable solution to the one-year risk horizon that is not recursive or circular.

1. The challenge: Solvency II and the Economic Balance Sheet

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1.1. Key terms

Best Estimate of Liabilities (BEL): Present value adjusted (discounted) sum of the means of the calendar year liability distributions. BEL(k) is the Best Estimate for the kth calendar year.

Market Value Margin (MVM): Present value adjusted (discounted) sum of the calendar year’s risk capital requirement multiplied by the spread. MVM(k) is the cost of risk capital for the kth calendar year. Also known as Risk Margins.

Technical Provision (TP): The sum of BEL and MVM. TP(k) is the sum of BEL(k) and MVM(k) for the kth calendar year. Also referred to as Fair Value of Liabilities.


Spread (s): The cost of funding $1 of risk capital above the risk free rate; expressed as percentage risk premium – stipulated by regulation to be 6%.

Value-at-Risk (V@R): The loss exceeding the best estimate at a specified percentile. V@R_{99.5} is the distress Value-at-Risk.

Risk Capital (RC): The capital required to meet losses at V@R_{99.5} and any additional capital to restore the economic balance sheet to fair value.

Solvency Capital Requirement (SCR): The risk capital required for the next calendar year.
1.2. Critical information

In order to compute Technical Provisions, Market Value Margins using the Cost of Capital approach, and the Solvency Capital Requirement for the one-year risk horizon for the aggregate of all long-tail LOBs (and each LOB), the following critical information is required:

- Probability distributions of the paid losses by calendar year \(k=1,\ldots,n\), and their correlations for each LOB and the aggregate of all LOBs where complete run-off is achieved at the ultimate calendar year \(n\).
- Probability distributions of the paid losses conditional on the first calendar year’s losses being at the 99.5th percentile; that is, the year is ‘in distress’ with a 1/200 year event.

Armed with the above distributions, any risk measure can be computed, including Value-at-Risk (V@R) for calendar year \(k\), for each LOB, and the aggregate of all LOBs.

1.3. Correlations between multiple LOBs

There are three types of correlations between LOBs: process (volatility) correlation, parameter correlation, and reserve distribution correlation. Process correlation is the correlation in the pure volatility component of the liabilities. This is measured after all trends have been accounted for. Parameter correlation can arise from the action of external effects, but is also induced via process correlation because estimation of model parameters depends on data subject to correlated random effects. Reserve correlations between calendar year liability streams are a function of process variability and parameter uncertainty; higher parameter uncertainty results in higher reserve correlations.

1.4. Correlations and risk diversification credit

Correlation should be measured from the data in order to determine each company’s unique interline correlation. Taking these into account results in alterations in parameter and volatility estimates and hence in reserve distributions. These effects cannot be replicated by the imposition of off-the-shelf correlation matrices or copulas. Correlation is an intrinsic component of a good model.

Modelling multiple Lines of Business simultaneously leads to significant aggregate risk diversification credit. The level of diversification is dependent on the correlations between the LOBs. Lines A and B have a process correlation of 0.52 and a reserve distribution correlation of 0.38. Individually the 99.5% Values-at-Risk (V@Rs) are $2.22M and $4.06M. If they were treated as uncorrelated the 99.5% V@R of the aggregate is $6.12M, but if the correlation (0.37) is correctly factored in, this figure increases to $6.81M. In this case, an accurate estimation of reserve distribution correlation leads to an 11% increase in risk funds over the assumption of full diversification.

1.5. The impact

Holding the incorrect amount of capital can be very costly especially with the additional requirements imposed by Solvency II. It is imperative that a company’s risk volatility characteristics are modelled accurately. Accurate modelling has two immediate benefits - (1) risk capital is accessed at the most optimal and cost-effective rate; (2) timely and critical financial information is provided regarding emerging trends enabling efficient management of liability risks. Correct treatment of correlations also leads to risk diversification of SCR and TP.

1.6. Causes of distress

If the model and associated forecast scenario are correct, then there are two potential causes of distress – extreme observations or high trends. In the probabilistic trend family (PTF/MPTF) modelling frameworks, distributions are fitted to describe trends and individual periods. These distributions therefore also describe extreme values. For instance, if a calendar year trend of 10%+ 2% is used for projection, a distress scenario may be a realised future calendar year trend of 20%.

Inaccurate modelling, however, could result in a company being in distress not due to a 1 in 200 year event, but rather because the projected probability distributions by calendar year and associated correlations were wrongly estimated.

1.7. Definition of Solvency Capital Requirement

The SCR for the one-year risk horizon is the distress Value-at-Risk for the first year plus the change \((A)\) in technical provisions in the subsequent years (suitably discounted), conditional on the first year being in distress. This definition satisfies the directives and advice provided by CEIOPS.

\[
\text{SCR} = \text{VaR}_{99.5\%}(1) + \Delta \text{TP}(2) + \Delta \text{TP}(3) + \ldots + \Delta \text{TP}(n);
\]

where \(n\) is the number of years until run-off.

The first year being in distress impacts the subsequent years the effect is measured by the \(\Delta \text{TP}\). Including the adjustments changes the estimates of SCR and MVM for the first year. Recursion is not required if only the first year in distress is considered.

Calculated in this way, the SCR is adequate to restore the balance sheet to a fair value of liabilities at the end of a distressed first year so that the portfolio can then be transferred or sold to a reinsurer. That is, the economic balance sheet has sufficient SCR and TP to sustain a first year in distress and be restored to its fair value at the beginning of the second year. We believe this formulation satisfies the Solvency II directives. However, it may be important to consider whether the receiver also subscribes to Solvency II; a solution for this case is also presented. See section 3.
2. The solution: One-year risk horizon

For the one-year risk horizon, Risk Capital is raised at the beginning of each year; risk capital is not part of the technical provision. The cost of raising the risk capital, the Market Value Margin or Risk Margin, is paid to the capital provider(s) at the end of each year along with any unused risk capital. In this solution, MVM is not used to pay claims as the risk capital is sufficient in the event of reaching the distress threshold; the transfer of funds if losses exceed this threshold is not defined.

The sum of the MVMs and Best Estimate of Liabilities (BELs) for each calendar year is the Technical Provision (also referred to as Fair Value of Liabilities). It is assumed that all capital, including the risk capital fund, is invested by the company at the risk free rate. In the event that this is not the case, the discount formulae for MVM require appropriate adjustment.

If the BEL is based on suitable assumptions, then we expect that in the long term, draws on the risk fund and the reserve surpluses will balance out to zero; the risk capital fund operates as a form of reinsurance. Distress years arise when reserves and risk capital are exhausted—expected to be a 1/200 year event if the estimates of BEL and risk capital are correct.

The distress scenarios are described by the model and do not imply the model was wrong. That said, a poor estimate of BEL could itself result in distress conditions being encountered which would not have been the case with a correct estimate.

Example: Uncorrelated future calendar years

If calendar years are uncorrelated, then the first year being in distress does not impact the distributions of subsequent paid losses liability streams. The $V@R_{99.5}$ for each calendar year is the assigned risk capital for those years; no revision after distress is needed. The relationship between the years is shown below followed by the flow of capital for the first year.
2.1. Correlation between future calendar years

Nominally, each future year has an allotted risk fund equal to V@R. Retaining the risk capital for each calendar year at the original V@R is naïve as it assumes that a distress event has no effect on distributions in subsequent years. It is implausible that a distress year would result in no updating of the model; a model incorporating parameter uncertainty would respond automatically to incoming data. That is, as data arrive in calendar time, the knowledge about the future parameters improve and estimates are incrementally adjusted according to the data observed.

Consider the following data displayed as at year end 2010 with projections to 2020. The parameter uncertainty is as indicated by the central grey bars while the variation contributed by the combination of parameter and process volatility is illustrated by the upper and lower red bars.

If the paid loss outcome for the first future year is known to be at the 99.5th percentile then the distributions for all subsequent years will be altered. The unconditional distributions must be replaced by the conditional distributions, where the conditioning event is the first year is at the 99.5th percentile. The technical provisions and risk capital raised at the beginning of each year will change according to these new distributions.

Consider the case for the data above where the next calendar year (2011) arrives at the 99.5th percentile. The new projected distributions of the future conditional on 2011 arriving at the 99.5th percentile are adjusted as illustrated below.

The new estimate of the trend is now substantially higher – as is the uncertainty associated with the trend. The process volatility drops since the forecast horizon is shorter.

In the example, the upper percentiles (process and parameter) increase slightly should the first year be in distress however the most significant revisions are to the lower percentiles.

Considering only the second year, the technical provision based on the conditional distribution will be higher than the technical provision based on the unconditional distribution. The adjustment required to restore the economic balance sheet to fair value should the first year be in distress is the difference between the two provisions: ΔTP(2).

Since the risk capital must be sufficient not only to cover the excess losses for the distress year (V@R99.5) but also to restore the balance sheet to fair value at the beginning of the second year, the risk capital should be supplemented by ΔTP(2). When the first year is in distress, this additional capital is allocated to restore technical provisions in the second year.

The same result applies to all future calendar years where capital is allocated from the first year’s risk capital to restore the provisions to fair value.
If the first year is in distress, then the conditional distributions for subsequent years change, resulting in higher technical provisions. The conditional distress V@R, MVM, and BEL for the second year are \( V@R(2|\xi) \), \( MVM(2|\xi) \) and \( BEL(2|\xi) \) respectively.

The capital we need to hold in the first year to restore the balance sheet from a distress situation is equal to the sum of differences between the conditional and unconditional MVM and BEL. As illustrated in the diagram above, \( \Delta BEL \) and \( \Delta MVM \) form part of the SCR in the first year and are to be discounted accordingly. Should the first year be in distress, only the MVM is returned to the capital provider, the \( \Delta TP \) is allocated to future years to restore the balance sheet.

\[
\begin{align*}
\Delta BEL(2) &= E[L|2|\xi] - E[L_2] * PV(0.5) \\
\Delta MVM(2) &= (MVM(2|\xi) - MVM(2)) * PV(1) \\
\Delta V@R(2) &= V@R(2|\xi) - V@R(2) \\
\Delta TP(2) &= \Delta BEL(2) + \Delta MVM(2) \\
SCR &= V@R_{99.5}(1) + \Delta TP(2)
\end{align*}
\]

where \( PV(k) = \frac{1}{1 + \text{risk free rate})^k} \) and \( \Delta TP(2) \) is \( (\Delta BEL(2) + \Delta MVM(2)) \) if this value is positive; otherwise \( \Delta TP(2) \) is zero.

**Example: Two year run-off (correlated)**

The general case is simply an extension of the two-year example where instead of adjusting the SCR by only the additional capital required for the second year adjustments for subsequent years are also incorporated. That is:

\[
\Delta TP = \Delta TP(2) + \Delta TP(3) + \ldots + \Delta TP(n)
\]

where \( n \) is the limit of run-off.

**Example: n year run-off (correlated)**

This changes MVM(1) and hence TP(1), however for \( k > 1 \), MVM(k) and BEL(k) are unchanged since in the event of the distress scenario we can draw on the risk fund for the amounts \( \Delta MVM(k) \) and \( \Delta BEL(k) \) to augment the MVM(k) and BEL(k) appropriately.
\[ \Delta \text{BEL}(k) = (E[L_k|\xi] - E[L_k]) \times PV(k-0.5) \]
\[ \Delta \text{MVM}(k) = (\text{MVM}(k|\xi) - \text{MVM}(k)) \times PV(k-1) \]
\[ \Delta \text{TP}(k) = \Delta \text{BEL}(k) + \Delta \text{MVM}(k) \]

where \( PV(k) = 1/(1 + \text{risk free rate})^k \) and \( \Delta \text{TP}(k) \) is \((\Delta \text{BEL}(k) + \Delta \text{MVM}(k)) \) if positive; otherwise \( \Delta \text{TP}(k) \) is zero.

Here \( \text{BEL}(2+) \) and \( \text{MVM}(2+) \) simply refers to the sum of the respective \( \text{BEL} \) and \( \text{MVM} \) allocated to future years after the first year.

### 2.2. If the first future year is in distress...

In summary, if the first future year is in distress, then the following events occur:

- \( \text{BEL}(1) \) and \( \text{V@R}_{99.5}(1) \) are exhausted (the losses incurred were at the 99.5% quantile).
- \( \text{MVM}(1) / PV(1) \) is returned to the capital providers and is not used to pay claims since \( \text{BEL}(1) \) and \( \text{V@R}_{99.5}(1) \) are sufficient to pay for claims. Total capital returned is: \( \text{SCR} \times (\text{spread} + \text{risk free rate}) \).
- The change in technical provision, \( \Delta \text{BEL}(k) \) and \( \Delta \text{MVM}(k) \), are assigned to the respective future calendar year \( \text{BEL}(k) \) and \( \text{MVM}(k) \) respectively.

It is emphasized that any return on \( \Delta \text{BEL}(k) \), \( \Delta \text{MVM}(k) \) in the first year is returned to the capital providers as part of the risk free rate calculated on the \( \text{SCR} \).

At the beginning of the second year the economic balance sheet has been restored to fair value after a distress scenario in the first year.
3. One-year risk horizon: multiple calendar years in distress

The SCR as calculated for one year in distress is adequate to restore the balance sheet to a fair value of liabilities at the end of a distressed first year such that the portfolio can then be transferred or sold to a reinsurer. In this section, we consider the solution where it is expected that fair value incorporates that the receiver also subscribes to Solvency II.

If the receiver also subscribes to Solvency II then the receiver must also hold sufficient capital for their next year being in distress (the second future calendar year) and being able to provide sufficient fair value to the receiver of the liabilities should they have to transfer them.

Since this naturally applies to the receiver, the result is that the analysis must consider a distress scenario for every year. The solution is not recursive if the transferrer only considers the situation where the future receiver is in distress and does not consider any adjustments to the risk capital fund to account for the multiple combinations of potential distress scenarios. That is, a single step procedure is performed where each future year is analysed as being in distress without conditioning on the losses in prior future years. Since Solvency II only refers to a one-year time horizon, the single-step restriction is reasonable.

Example: Three year run-off (correlated)

In a three year run-off there are two distress years to consider conditionally – the first and second future calendar years. The new terms in the diagram are $\Delta_2 MVM$ and $\Delta_2 V@R$. The additional risk capital required in the third year should the second year be in distress is $\Delta_2 V@R$. The cost of raising this capital is $\Delta_2 MVM$. For the purpose of this calculation, $\Delta_1 MVM$ and $\Delta_2 MVM$ are considered additive – an adjustment for year 1 and year 2 both being explicitly in distress is not included.

For this solution the steps are as follows:

1. Simulate all future calendar years – these are the unconditional distributions
2. Select simulations around the 99.5th percentile for the first year from the unconditional distribution (there are multiple ways to reach distress and it is unwise to condition on just one simulation). Simulate the conditional distributions arising for all subsequent future years to run-off.
3. From the conditional simulations in (2), calculate the adjustments to technical provisions required for subsequent years should the prior year be in distress. Add the cost of the sum of the adjustments, suitably discounted, to the MVM for this year.
4. Repeat (2-3) for the second year through to run-off – 1 (the last calendar year in run-off has no future calendar year to condition for).

The selection of the distress samples from the unconditional distributions eliminates recursion.
4. ICRFS-Plus™, Solvency II and IFRS 4 Phase II

The proposed Solvency II and IFRS 4 phase II standards focus on risk measures to be introduced into insurance regulation/insurance accounting respectively.

IFRS 4 phase II has diverged from the proposed Solvency II standard in the Exposure Draft (2010) as the notion of ‘transfer of liabilities exit value’ (liabilities fair value) has been de-emphasised. However, similar risk measure concepts still exist in both proposed standards. Both standards require accurate estimation of cash flow distributions – not just means, but a measure of risk adjustment (market value margin) which are then incorporated into the economic balance sheet.

Risk adjustment techniques, specified in the IFRS 4 phase II Exposure Draft (2010), require the measuring of probability distributions by calendar year of the underlying cash flows. Cost of capital, Conditional Tail Expectation and confidence levels (V@R) necessitate models that project distributions by calendar year.

The Probabilistic Trend Family (PTF) and Multiple Probabilistic Trend Family (MPTF) modelling frameworks within ICRFS-Plus™ provide the distributions required to compute the risk adjustment measures. In MPTF, a single composite model for multiple LOBs, identified from the data, succinctly describes the volatility in each LOB and the correlation structure between them. Easily interpretable parameters forecast lognormal distributions for each cell for each LOB. From these parameters, distributions and correlations are computed by accident year, calendar year, and total for each LOB and the aggregate of the LOBs.

Since Solvency II calculations also require the above distributions by calendar year, and both impact the economic balance sheet, ICRFS-Plus™ is the ideal tool to construct internal models that meet both Solvency II and IFRS 4 phase II requirements for long-tail liabilities; calculation of the core components can be sourced from the same model. The joint use of ICRFS-Plus™ internal models for both applications implicitly reduces implementation costs.

4.1. Fungibility and ring fenced funds

In IFRS 4 phase II Exposure Draft (2010), BC119, the level of aggregation for risk margins are considered, with the recommendation of the board being to determine risk adjustments at the level of individual portfolios. The question of aggregation over portfolios, or Lines of Business, is important for determining the risk fund as it is unlikely to be appropriate to assume all LOBs within an entity are fully fungible (that is, an excess (loss less than the reserved mean) in one LOB is fully available to cover a deficit (loss greater than the reserve mean) in another LOB).

Ring fenced funds where excesses in a group of LOBs do not supplement losses in another LOB or have other restrictions are discussed in QIS 5. A ring fenced fund as referred to in the level 1 text arises as a result of an arrangement where:

a) There is a barrier to the sharing of profits/losses arising from different parts of the undertaking’s business leading to a reduction in the pooling/diversification related to that ring fenced fund; or

b) Own funds (restricted own funds) can only be used to cover losses on a defined portion of the undertaking’s (re)insurance portfolio or with respect to particular policyholders or in relation to particular risks such that those restricted own funds are only capable of fulfilling the criteria in Article 93(1) (a) and/or (b) in respect of that defined portion of the portfolio, or with respect to those policyholders or those risks; or

c) Both a) and b) apply (1.2)

The determination of risk adjustments, or risk margins, at the LOB level (as recommended by the IFRS board), however, negates one of the fundamental principles of insurance – namely risk pooling, including of the risk fund. Calculating the risk margin at the LOB level prevents diversification credit of the risk fund since each adjustment is applied independently. There are solutions which include diversification credit of the risk fund, while not diversifying excesses arising from losses below the means. Consider the structure where a risk fund is pooled for all LOBs written. Under this structure, a number of options for fungibility, or lack thereof, can be considered.

1) Full fungibility; an excess in one line can supplement a deficit in another line.

2) No fungibility; excesses in a line are retained in that line and do not supplement the risk fund.

3) Partial fungibility; excesses in a portfolio are fungible within the LOBs comprising the portfolio, but not between portfolios. Here a portfolio refers to a group of LOBs (possibly in a cluster) which are ring fenced as a whole.

The options can be broken down further since excesses and deficits arrive in calendar time. Applying the above reasoning, the following scenarios can be considered:

1) No sharing of excesses by calendar year

   In this case funds are treated as if already allocated to each calendar year according to the forecast estimates. If the losses for the next calendar year exceed the estimated mean, the difference is made up by drawing from the risk pool. If the losses are below the allocated reserve the difference is released as profit; the excess amounts are not rolled over into the reserve fund.

2) Buffered diversification by calendar year

   The accumulated excess from previous calendar years is retained as a buffer fund to be drawn on before calls are made on the risk fund. In this case pooling occurs only in relation to past years (drawing on a future excess is not permitted). The retention of funds allocated to future years retains a higher priority than drawing on the risk fund, so that funds allocated to future calendar years cannot be accessed.

   The calendar year scenarios are readily extended to multiple LOBs where fungibility between LOBs is limited. The various combinations provide for a broad range of preserving policies, and can be adapted to respond to different regulatory or prudential regimes. Present value accounting can also be adapted to the timing inherent in each set-up.

   Should excesses supplement future losses, including across fungible LOBs or calendar years, or can they be released as profit? Using ICRFS-Plus™ models, forecast scenarios, and simulations, all of the above situations can be considered.
Example: $L_A$ and $L_B$
Consider two LOBs: $L_A$ and $L_B$
Case 1: The aggregate loss distribution, $L_A + L_B$, assumes $L_A$ and $L_B$ are fully fungible.
Case 2: If $L_A$ and $L_B$ are restricted such that the surplus arising from any loss less than the mean of $L_A$ or $L_B$ are retained by $L_A$ or $L_B$ respectively, then the aggregate distribution becomes:

$$L^*_A + L^*_B$$

where:

$$L_A = \text{Max}(L_A, \text{mean}(L_A))$$

$$L_B = \text{Max}(L_B, \text{mean}(L_B))$$

The above implicitly implies that any loss less than the mean of either LOB does not contribute to the risk pool, and can be managed according to company policy (release or retention) without any loss of risk cover. This represents a loss of diversification credit for writing multiple lines.

In the case of two independent lines, the loss of diversification can be in the order of 5-10%. If the two lines are positive-correlated, then the loss of diversification is lower since it is more likely that both LOBs are above the mean when draws on the risk fund are made.

5. Ultimate year risk horizon, risk capital, and market value margin

In the ultimate year risk horizon paradigm, risk capital is sufficient to cover adverse development up to a certain (eg 99.5%) quantile in the reserve distribution for the whole run-off period. The entire risk capital, that is, Value-at-Risk for the aggregate reserve ($\text{V@R}[\text{Aggregate}] = \text{V@R}[1]$) is raised at inception and unused capital released back to the capital provider(s) at the end of each calendar year.

At the beginning of calendar year $k$ (for $k>2$) the amount of risk capital retained for the remaining run-off period is $\text{V@R}[k]$. The amount of risk capital released back to the capital providers at the end of calendar year $k$ (for $k=1, \ldots, n$) depends on the total loss in year $k$, the mean loss for year $k$, risk capital $\text{V@R}[k]$ retained at the beginning of year $k$, and risk capital $\text{V@R}[k+1]$ retained at the beginning of year $k+1$.

The principal difference between the one-year risk horizon and the ultimate year risk horizon in respect of computing risk capital and MVM is when capital is raised from the capital providers not when capital is returned to the capital providers.

6. CASE STUDIES

6.1. Diversification of SCR and MVM based on an internal ICRFS-Plus™ model

In the following discussion we consider six LOBs (labelled LOB 1 through LOB 6) with the following internal ICRFS-Plus™ model. The displays highlight the trends (clockwise from top left) in the development, accident, volatility around the development trends, and trends in the calendar direction.
Solvency II

LOB 4 is the most volatile of the LOBs due to very unstable calendar year trends. The high uncertainty in the parameters results in high calendar year correlations which in turn impact risk capital requirements in future calendar years. The comparative volatility for the LOBs and for the aggregate is illustrated in the following displays.

The two largest LOBs, LOB 1 and LOB 3, take the bulk of the mean reserve, but the smaller LOBs, LOB 4 and LOB 5, take a larger percentage of total risk capital (relative to their mean). That said, almost all the risk capital allocation for the next calendar year (2010), is also primarily allocated to LOB 1 and LOB 3! The other lines do not require risk capital till later in the run-off, thus Solvency II one-year risk horizon (one year in distress) capital requirements are effectively driven by these two LOBs only. This result emphasises the importance of not only having the right mean and standard deviation estimates, but also the correlations to be able to allocate capital to each calendar year correctly.

The total capital required for the aggregate of the six lines is almost the same as the mean of the undiscounted reserves.

If LOB 4 was written on its own (right), significant additional capital is required compared to the undiscounted reserves. This additional capital is principally allocated to ΔTP, capital needed to restore the balance sheet should the next year be ‘in distress’. The calendar trend is very sensitive to new observations which, if high, result in a higher estimate of the calendar year trend requiring substantial revision of future calendar year BELs (the probability of being in distress as a result of a poor estimate of the calendar year trend is high). The risk of LOB 4 is not a result of process variability but parameter uncertainty.

The charts show the allocation of capital at inception and for the first year to the various components: BEL, MVM, V@R(2010) and ΔTP. For the aggregate (left), the display is dominated by BEL – the other components do not form a large percentage of the total capital required. For LOB 4 (right), ΔTP is almost as large as BEL – a result of the high calendar year correlations.

The allocation of technical provisions to each future calendar year given the first year is in distress is very different for the aggregate of the six lines (left) versus LOB 4 (right).

Both V@R and BEL taper off quickly for the aggregate, although the later years have proportionally higher V@R. For LOB4, however, the V@R increases rapidly by calendar year and does not decrease till the last few calendar years. Similarly, the BEL increase before slowly decreasing. V@R is a huge proportion of the mean for most calendar years for LOB 4.
6.2. Multiple years in distress

If multiple years are considered as being potentially ‘in distress’ then more risk capital allocated to each future year is required in order to restore subsequent economic balance sheets. The capital requirements and adjustments reflect the driver of distress in each calendar year. Risk capital contributes significantly to the total capital requirements as the calendar year’s progress. Furthermore, the proportion of the risk capital by LOB varies; the proportions are indicative of the expectations of distress over time. In 2010, the most likely causes of distress are LOB 3 and LOB 1 respectively, whereas by 2016, the most likely cause of distress is LOB 4.

From 2013 the VaR is a low percentage of risk capital and the majority of the risk capital is required to adjust future BELs. This is primarily reflective of the high calendar year correlations in LOB 4 as this line becomes increasingly prominent over the emerging liability stream.

6.3. Diversification and fungibility

The changes in risk capital required for differing degrees of fungibility (or ring fencing) are detailed as follows. Full fungibility has the lowest risk capital requirement, with a low additional increase in risk capital related to a buffered fund. Holding all LOBs as individual entities and only pooling the risk capital incurs the largest penalty (as could be expected).

<table>
<thead>
<tr>
<th>LOB 1 through LOB 6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>One year in distress (2010)</strong></td>
</tr>
<tr>
<td>Fungibility</td>
</tr>
<tr>
<td>Across LOB, Forward</td>
</tr>
<tr>
<td>Across LOB</td>
</tr>
<tr>
<td>Not Across LOB, Forward</td>
</tr>
<tr>
<td>None</td>
</tr>
</tbody>
</table>

| **Multiple years in distress (2010–2029)** |
| Fungibility | SCR | MVM | % change in risk capital | % change in cost of capital |
| Across LOB, Forward | 68,660 | 26,799 | - | - |
| Across LOB | 68,546 | 26,935 | -0.17% | 0.50% |
| Not Across LOB, Forward | 69,206 | 27,108 | 0.79% | 1.14% |
| None | 69,131 | 27,399 | 6.68% | 2.15% |

It should be noted, however, there may be significant variability in SCR irrespective of the fungibility selection if fewer simulations are conducted; MVM generally is expected to be more consistent.

Not all results are intuitive. For instance, for multiple years in distress, not Across LOB, forward has a significantly lower cost of capital. This is a result of LOB 4 not being a significant contributor to distress in the next few years – thus giving a greater probability of accumulating a surplus which then supplements subsequent distress scenarios where LOB 4 is highly likely to be in distress. The net result is a lower cost of capital.
7. Consistency of Solvency II risk measures and prior accident year ultimates on updating

The Solvency Capital raised each year on updating, should not be subject to major shocks due to model errors.

Solvency II risk measures and estimates of prior year ultimates will be statistically consistent on updating from year to year if the forecast assumptions for the next calendar year play out and subsequent forward assumptions are statistically equivalent.

For instance, consider a company that writes the same mix of risks each year with the same exposure level each accident year. Risk capital is raised from the same provider with no changes in required return (risk free or spread).

Suppose calendar year inflation is stable and is 10%±2% in the most recent years. For reserves to be computed accurately, the company assumes this 10%±2% trend going forward along the calendar years.

In this scenario, the following applies:

- Each year the company needs to increase its total reserve by at least 10%.
- Each year the company needs to increase its premium (price) by at least 10%.
- The ultimates for prior accident years will remain consistent with each increase in total reserves.

It is only possible to maintain consistency if all the assumptions made in forecasting are consistent, explicit and auditable. The Probabilistic Trend Family modelling frameworks, both single and multiple, provide the basis for such consistency.

Comparing the statistics and risk measures above we find that there is consistency between the two valuation periods - 2008 and 2009. The model parameters are statistically the same (calendar parameter excerpts shown).

The means are consistent (the projected 2010 mean increases by around 4M - which is in line with a 9%± increase (given the realisation for the 2009 calendar year).

Note the estimates of the ultimates between 2008 and 2009. In 2008, the mean ultimate for accident year 2008 is 64.98M. In 2009, the estimate of the same (2008) accident year’s ultimate is 66.2M. These estimates are statistically consistent as the estimates are mean ultimates. The expected variation in 2008 of the mean ultimate (2008) is 4.3M. The new estimate of the ultimate in 2009 is well within one standard deviation. Similarly, the 2007 estimates of 69.8M and 71.6M for 2008 and 2009 year end are statistically consistent.
The two columns on the right of the accident year summary tables (previous page) show the expected change conditional on the next year’s data.

The first column, Std Dev | Data, shows the expected new standard deviations for next year’s model conditional that the next year’s data follows the projected model assumptions. Similarly, the second column shows the expected change in Ultimate given the next year’s data follows the projected model assumptions. This allows us to check for consistency between ultimates when the next year’s data are included in the model.

The calculation of risk capital is also consistent between years; the SCR requirement is essentially the same for both valuation periods. Note a discount rate of 4% and a spread of 6% are applied.

- Solvency Capital Requirements remain statistically the same (the risk and cost of capital have not changed).

The SCR estimates of 27.9M and 28.3M are almost identical for 2008 and 2009 respectively. The estimates of BEL and MVM increase from 2008 to 2009 but this is due to the 9.2% increase in calendar trend. As mentioned above, consistency depends on whether the predicted forecast assumptions for the next year and following a) occur for the next year and b) that the same forecasting assumptions used previously for the following years are applied in the subsequent forecast. Any change in assumptions will make prior year ultimates on updating inconsistent.