Pricing: Segments, Layers and Reinsurance
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1. Introduction and Summary

Pricing, reserving and reinsurance of long tail liabilities are connected issues. All relate to modelling loss development arrays. In order to calculate the required measures accurately, it is important to use a modelling framework capable of extracting all the pertinent information from the data. Any assumptions made by the model or future forecasting scenarios should be transparent, auditable and verifiable.

Two modelling frameworks that satisfy these criteria are briefly described. These frameworks are both included in ICRFS-Plus™ - the only commercially available, interactive, integrated modelling software providing high powered analysis at unparalleled speed.

Within these frameworks, reserve risk, future accident (underwriting) year risk and the combined (reserving and future accident year) risk can be meaningfully evaluated. Combined risk is of the most import given that typically there is a significant proportion of renewable business. The two projections, reserve and underwriting, are not independent – it is important to consider the benefit of risk diversification of future underwriting periods with the reserve risk.

Case studies are provided which illustrate creative solutions to pricing and reinsurance applications. Examples include:

- Pricing future underwriting years - single and multiple segments;
- Assessing capital efficiency of outward reinsurance;
- Selecting optimal layers as outward reinsurance structures;
- Pricing high severity / low frequency layers;
- Assessing prospective and retrospective adverse development cover;
- Protection against adverse calendar year trends.

The selected examples highlight the effectiveness of ICRFS-Plus™ as a tool for assessing and pricing risk. All metrics are derived from the data where future assumptions are under the actuaries control. Additional distribution assumptions are not required. Note that loss distributions can be assessed from the perspective of the cedant or inward reinsurer simultaneously.

1.1. Modelling frameworks

In the Probabilistic Trend Family (PTF) modelling framework an optimal model is identified (designed) that captures (adjusts for) the trends in the three directions: development period, accident period and calendar period; and the distributions of the volatilities about the trends.

Single composite models are designed in the Multiple Probabilistic Trend Family (MPTF) modelling framework for multiple segments or LOBs. These models are built from the identified PTF models and include the correlation structure between the segments or LOBs.

The diagram below depicts the three directions with arguments d, w and t. Since \( t = w + d \), it is axiomatic that any calendar year trend projects onto the development year and accident year directions.

The PTF and MPTF models uniquely address the features within the data – there is no set algorithm specifying a priori where trends (or volatility) changes occur. Trends are fully interpretable and are able to be related to events occurring in the business (whether driven by internal or external drivers). Each cell of the loss development array is related by the trend structure on a log scale. Correlations between the distributions, whether within or between segments (or LOBs), are incorporated directly in the model.

Random (loss development array) samples from the identified MPTF model are indistinguishable from the real loss development arrays in respect of statistical features. This test mitigates model specification error.
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An identified model and forecast scenario projects log-normal distributions and their correlations (within segments; and, for MPTF, between segments) for each cell. Future trends: development year, calendar year and accident year (for pricing); and process volatility about the trends are completely within the actuary’s control.

In this way, reserving, pricing, and reinsurance calculations can be conducted in a transparent manner with appropriate consideration of risk. For instance, if environmental conditions suggest a bad year for storms in the next underwriting period, then this expectation can be included in the projection by increasing the accident year level parameter and/or volatility by an appropriate magnitude. Alternatively, legislative changes may emerge as calendar year effects. If the actuary has information to suggest changes in forecast assumptions, then future trends can be monitored and revised accordingly.

In these modelling frameworks, probability distributions and their correlations are specified by the models and forecast scenarios for each cell. This attribute allows the design of creative solutions to estimate any statistical metric from the projected distributions. The same single composite model can be used for reserving, pricing, assessing risk diversification credit, evaluating inward/outward reinsurance programs, calculating Solvency II one year risk horizon metrics and much more! Further details can be found in other promotional material and on Insureware’s website (www.insureware.com).

1.2. Case studies

The remainder of this document comprises a series of case studies illustrating the depth and breadth of applications of ICRFS-Plus™ to long-tail liability risk management.

Section 2: Pricing future underwriting years

Various strategies for pricing future underwriting years are discussed. Three examples serve to illustrate methods for pricing single segments, pricing multiple segments, and strategically amortising premium changes to reach management targets. Loss distribution metrics, including and T-V@R with: Tail-Value-at-Risk (T-V@R), are summarised in preparation for the subsequent sections.

Section 3: Assessing capital efficiency of outward reinsurance

Techniques to assess the capital efficiency of existing outward reinsurance, including E&O D&O segments, are demonstrated via two examples.

Section 4: Selecting optimal layers as outward reinsurance structures

Several examples of layers and the methods of selecting optimal, capital efficient, outward reinsurance structures are presented. The reinsurer loss distributions are also defined so efficiency can be considered from the point of view of the cedant or inward reinsurer.

Section 5: Pricing high severity / low frequency layers

An example of pricing the layer 5M xs 5M using the data limited to 5M and the data limited to 10M is described. The 5M xs 5M layer is replete with zeroes and pricing this layer on its own would be guesswork. The strategies encompassed in this section show how sparse upper layers can be projected utilising information in the ground-up or other limited data.

Section 6: Assessing prospective and retrospective adverse development cover

Creative solutions are presented which illustrate the assessment of adverse development cover both retrospectively and prospectively. The analysis is performed on multiple lines, an individual line, and the combination of different adverse development structures by segment.

Section 7: Adverse development cover as protection against adverse calendar year trends

A real-life example shows calendar year trends where a high 35%+-4% trend was observed in the data previously. Adverse development cover is presented as a strategy for protection should this trend arise in the immediate future.
2. Pricing future underwriting years

2.1. Future underwriting year forecast scenarios

Two examples for single lines of business (LOB 4 and PM) are provided to illustrate the level of control over future trend and volatility assumptions the actuary has in the ICRFS-Plus™ probabilistic modelling frameworks.

- The same model should be used to project reserve and future periods;
- The effect of the assumptions on pricing future underwriting years are considered;
- Trends in loss ratios are examined and linked with calendar year trends;
- Premiums are set based on target expected loss ratios in combination with future forecast assumptions.

As probabilistic models relate every cell, trends can be projected in any time direction including extending beyond the boundaries of the loss data into future development or underwriting periods.

Control over future assumptions, particularly regarding future underwriting years, is essential. Future assumptions should be explicit so that their effect on pricing and reserve risk can be well understood.

In ICRFS-Plus™ the user has full control over all forecast assumptions within the probabilistic trend family modelling frameworks.

The model display to the right shows the measured trends for LOB 4 in each direction (development, accident, and calendar) along with the process volatility (lower right display) variance of the normal distributions by development period. The process volatility is high for this line and so is the parameter uncertainty by calendar year. The development trend does not begin decaying until seven years. That is, it takes seven years before the incremental paid losses start decreasing by development period.

The volatility in the calendar year trends (which measure the combination of both economic and social inflation) are of primary importance when assessing the future underwriting year’s loss distribution. Exposure and accident level related risk assumptions for the future underwriting year are controlled via the alpha parameter.

In the following example we show the relevant windows to control the future forecast assumptions of the trends in the three directions. Similar control is also provided for the process volatility but is not included here.

Changes assumed by the actuary in this forecast are:

- The calendar year trends change from 14%+3.62% (the trend since 2005) to 21%+4% for the next two years, 14%+3.62% for the following four years, and 7%+1% thereafter.
- The accident year level for the next underwriting year is expected to increase by 10% (on average) versus the previous (2009) level. A high level of uncertainty (5%) is associated with this increase.
Although the forecast is extended for two periods in the development direction, no changes were made to the development year trend. Forecast scenarios can readily be compared to evaluate economic risk, exposure risk, and adverse development. Future underwriting year risk, reserve risk, and combined (future + reserve) risk, can all jointly be assessed under varying scenario conditions.

For this line, earned premium is low resulting in high loss ratios. The mean loss ratios are exceeding 100% from 2004 onward. They are also exhibiting a positive trend as the losses are increasing faster than earned premium. The calendar year trend is projecting along the accident years (and development years) and is increasing at a faster rate than the earned premium. Due to the high volatility in the line (including parameter uncertainty), the standard deviations of the loss ratios are also very high.

This would be of primary concern for pricing as this implies that other lines written are expected to compensate for this line’s premium being too low. If the premium for 2010 is at the same level as 2009, then the expected loss ratio is 253% for this scenario.

2.1.1. Primary Motor
Consider the following Primary Motor line. The trends in the three directions along with process volatility are shown below.
For this example, the increase in losses for 2009–2010 is attributable to bad weather conditions. Furthermore, the pricing actuary has information forecasting subsequent deterioration for the 2012 year. The scenario for the future underwriting year can be revised as follows:

Above, the same 9.93% + 6.23% increase is assumed to occur in the next underwriting year. No changes are made to the trend assumptions in the other two directions; nor are any changes made in respective to volatility.

All trend assumptions made for the future can be related to the trends measured in the data. Transparent trend assumptions can aid discussions with shareholders, regulators, and auditors.

Lognormal distributions are projected for each cell in the underwriting year. The distributions are correlated via the trend structure. All the trend and volatility assumptions driving these distributions are known and can be related to the data and business knowledge.

The earned premium, in order to maintain the same expected loss ratio as the previous year, requires an increase of 59% over the earned premium in 2010. For comparison, the earned premium increased by 78% for the corresponding increase in 2009–2010.

The pricing actuary can be more / less aggressive in pricing according to business policy and current market conditions. The most important attribute is that all appropriate assumptions can be varied and clearly evaluated. Discussions regarding future assumptions for models in the PTF/MPTF modelling frameworks are informative and aid in presentation to senior management, as compared to arguments over link ratios; for example, whether a ratio should be 1.15 instead of 1.1?
2.2. Multiple lines

A single composite model can be used to jointly project future underwriting period(s) for multiple Lines of Business. This example considers the task of pricing six LOBs.

Management expected loss ratio targets are considered as well as amortisation to reach these targets; Risk margins are included as a component of the premium as an example of this business strategy; The effect of risk diversification of the future underwriting year with the reserve periods is examined.

Reserve distribution correlation (*) is usually very low between reserve and underwriting risk. Any correlations between future and reserve periods are driven by common parameters. Common parameters are a further reason not to separate the reserve and underwriting calculations.

(*) See also “Understanding Correlations” by Insureware.

This section illustrates a number of concepts:

- Pricing multiple Lines of Business, segments, or layers simultaneously;
- Including risk capital / margins when considering pricing; and
- Amortising premium requirements to reach management targets (of expected loss ratios).

The following results are for the writing of six LOBs (the model displays are presented in section 6.1) for the next underwriting year assuming no change in exposure (for simplicity).

Loss ratios above are based on the premium for the previous year (2009) and are unadjusted. If left unchanged, some lines are expected to run at, or close to, an overall loss (LOB 1 and LOB 4 particularly).

The corresponding table for the reserve component is displayed below. For the sake of this example, the Expected Loss Ratios (ELR) are considered the long-term performance targets desired by management.

We observe:

- LOB 1 is currently aggressively priced and the premium should be increased over time to return the ELR for this LOB to the long-term target of 47%;
- Premium for LOB 2 can be increased to maintain the long-term target;
- Premium for LOB 3 can be increased to maintain the long-term target;
- LOB 4 is extremely volatile and premium should be increased according to the desired risk level. As the volume in LOB 4 is low compared to the other lines, minimal changes may be satisfactory in the short term (this is the application we apply here);
- LOB 4 and LOB 6 are both historically good performers but, to gain market share, the ELR has been allowed to increase to center between 55%~60% - this strategy is to continue for 2010;
- The largest line, LOB 3, has the most flexibility for adjustment to offset lower premiums in other lines.
Applying the above considerations, with no provision for risk margins or discounting, the target premium levels for the six segments are as follows:

The business strategy is unlikely to accommodate radical changes in premium - and increasing premium for LOB 4 by 52% for no change in exposure would constitute a major increase. To achieve the desired loss ratio (in aggregate), an increase in premium of at least 5% is sufficient. The increases (decreases) can then be allocated to each line based on market strategy and in combination with other risk factors (including risk capital and premium risk).

The mean and volatility (represented by the CVs) are shown for the future underwriting year (2010) below. The largest lines by reserve mean are LOB 3 and LOB 1, whereas the riskiest lines are LOB 4 and LOB 5. The latter lines are small relative to the other segments.

The distribution of the total losses for the future underwriting year (2010) for the aggregate of the six lines is displayed above. The total mean is at the 52% quantile (percentile). If premium was raised to cover the future underwriting year’s mean only, then losses up to 100M are at risk (at the 99.95th percentile). Risk appetite will depend on a number of factors. Here the business strategy is to price the undiscounted premium at the 75th percentile of the total losses for the next underwriting year. The losses at the 75th percentile (and others) are displayed below. The quantiles are also an estimate and the associated uncertainty can be calculated.
Pricing

Risk capital (in this case V@R at 75%) as a percentage of mean - illustrates the riskiness of the lines. The total risk capital is allocated to each line based on a variance-covariance formula; basically, in proportion to each line's contribution to the total volatility.

The revised table below shows the target premium levels for the six segments where risk capital allocation at the 75th percentile is incorporated in the ultimate.

2.2.1 V@R versus T-V@R
The Value-at-Risk at the 75th percentile (V@R75) is the capital that would be lost should the mean be reserved and the actual losses come in at the 75th percentile. The Tail-Value-at-Risk (T-V@R) is the average capital that would be lost should the mean be reserved and the actual losses exceed the specified percentile. In this respect, V@R is the minimum loss of capital given a percentile is reached; thus T-V@R is greater than V@R.

The lines receiving the most risk capital are the largest lines - LOB 1 and LOB 3 respectively. Premium raised for LOB 3 (and the currently good loss ratio) likely reflects the higher level of risk capital allocated to this line. LOB 3 provides a good diversification buffer for the more risky lines (LOB 4 is priced conservatively in the scenario considered for the next underwriting year).
2.2.2 Reserve Risk, underwriting risk, and combined (underwriting + reserve) risk

The above procedure of analysing the risk capital allocated to the future underwriting year for the six segments treats the future underwriting year on its own and does not consider diversification credit with the reserve distribution. Combined risk is always less than the sum of future plus reserve risk. In practice, the diversification gained by considering the combination of future underwriting year risk and reserve risk jointly is a significant factor.

For the 75th percentile, the gain in diversification benefit for the future underwriting year is significant. The total reserve risk capital (V@R) at 75% is 21.5M (left), for the future underwriting year, the risk capital (V@R) at 75% is 18.3M (center). Treating the two risk capital calculations separately results in a total risk capital calculation of 39.8M. When analysing the joint distributions from the reserve + future underwriting year, the total risk capital (V@R) at 75% is 34.9M (bottom). The reduction in the total required risk capital of 4.9M can be used to offset the risk capital margin for the premium in the future underwriting year (since premium has already been raised for the reserve period).

If this methodology is applied, the premium only needs to be increased by 9% to accomplish the same management targets.

2.3 Key learnings

- Projection for reserves and future underwriting years should use the same model;
- In the examples provided, the forecast scenarios for future underwriting years use the same calendar year trend assumptions as for the reserve component. This is not essential;
- Trends in mean loss-ratios provide timely information regarding premiums and the level of risk taken on by the insurer;
- Premiums can be amortised according to management requirements and business strategy;
- Premiums can also include risk margins in addition to the best estimate;
- Combined risk (future underwriting year + reserve) is less than the sum of the two risk components.
3. Assessing current outward reinsurance

3.1. Case study: E&O D&O

Outward reinsurance can be evaluated for capital efficiency retrospectively. Additional cover can also be evaluated. We present a case study involving E&O and D&O where we have Gross and Net of Reinsurance data.

- Model trend structure is very similar between Gross and Net of Reinsurance data;
- Process (volatility) correlation is very high since Net of Reinsurance is a subset of Gross;
- The current outward reinsurance is capital efficient for the cedant;
- The capital efficiency of the reinsurer is also evaluated.

This study illustrates the effectiveness of outward reinsurance in respect of capital efficiency and reduction of volatility in the losses Net of Reinsurance for the cedant.

The Multiple Probabilistic Trend Family (MPTF) modelling framework is used to analyse these segments jointly. The two model displays for the E&O D&O data, Gross and Net of Reinsurance respectively, are shown below.

The model trend structure is the same as one would expect (except for the early accident years). Calendar and development parameters are structurally almost identical; parameter changes occur at the same time points. This feature of the data indicates common calendar year drivers for the two segments – exactly what one expects from Gross and Net of Reinsurance. The calendar year trend is higher in the Gross than the Net of Reinsurance, so a cursory conclusion suggests inflation is higher in the Gross than the Net of Reinsurance. However, a closer examination reveals that the development trends are higher in the Net of Reinsurance than the Gross – the projected trends (sum of all directions) are very similar in the two segments. The key difference is in the process volatility; the Net of Reinsurance has significantly lower process volatility (lower right displays). This is the reason the Net of Reinsurance is successful – the more volatile claims are the ones being ceded to the reinsurer.

The process (volatility) correlation of 0.843 is not used in the models above (to ensure repeated measurements are not used) to estimate model parameters or in forecasting.

What does a high process correlation (between two sets of normally distributed residuals) imply? Namely, that the two segments behave in a similar fashion when it comes to observations arriving above or below the fitted trend lines. Process correlation is always relative to a fitted model.
The above displays show the trace line for calendar year 2006 (the last calendar year), for the same corresponding accident years. A trace line connects residuals from the same period (here calendar year 2006) making it easy to find the corresponding residuals when examining another direction. The shape of the trace line is very similar – when residuals are above the line in Gross they are extremely likely to also be above the line in the Net of Reinsurance – similarly points below the line (in fact for this trace there is a one-to-one correspondence). This is expected as Net or Reinsurance is a part of Gross.

The forecast summary by accident year, with CVs highlighted, confirms the observation made above that the Net of Reinsurance is more capital efficient. The CVs are consistently lower for the Net of Reinsurance; this is a sign of capital efficient reinsurance for the cedant. This is confirmed by simulating from the projected log-normal distributions for each cell (and their correlations) to obtain the complete distribution of aggregates below.

Similar conclusions can be drawn from the distribution graphs above for the Gross (left) and Net of Reinsurance (right) respectively. The Gross projections are far more skewed than the Net; and only the Kernel provides a good fit to the Gross projections.
3.1.1. Risk capital efficiency

The comparative capital efficiency for the two segments, Gross and Net of Reinsurance, shows that in terms capital efficiency, the reinsurance is working for the cedant. The Gross data requires significantly more risk capital (as a percentage of the mean), than the Net of Reinsurance.

From the Gross and Net of Reinsurance data we can also calculate (approximately) the capital efficiency of the reinsurer by calculating the difference between the Gross and Net of Reinsurance forecasts for each cell.

The risk capital efficiency of the reinsurer is very poor with the required risk capital being 89% of the mean. Note that the correlations between Gross and Net of Reinsurance were taken into account when calculating this figure at the 95th percentile.
3.2. Case study: FAC ENG

This case study illustrates current outward reinsurance that is not capital efficient from the perspective of the cedant. Model trend structure is very similar between Gross and Net of Reinsurance data;
Gross data calendar year trends are growing whereas Net of Reinsurance trends are not;
The process volatility is significantly higher in the Net of Reinsurance data.
The reinsurance seems to be effective in that the Gross data are growing whereas the Net of Reinsurance data are stable, however the higher volatility in the Net of Reinsurance data results in no gain in capital efficiency.

The Multiple Probabilistic Trend Family (MPTF) modelling framework is used to analyse the Gross and Net of Reinsurance data for FAC ENG jointly. The two model displays for the two segments are shown below. The red bar indicates common parameters between the two sections. Note the presence of a calendar year trend in the Gross data which is not present in the Net of Reinsurance. The same exposure base is used for both Gross and Net of Reinsurance eliminating any differences in trends arising from exposure.

The difference in the models is critical for the first two development year periods (where the constraint exists), but has significantly less impact in the later development year periods as the Net of Reinsurance decay is not as strong as the Gross thus cancelling out the calendar effect in the development direction. The difference is only 1.6% and is within the standard error of both parameters.
The calendar year trend projects onto both development year and accident year directions. It is in the accident year direction (left) that the growth occurs in the Gross (upper lines of each colour) but not in the Net of Reinsurance (lower lines of each colour). This is good in the sense that it is the part that is growing that is being ceded, however the outward reinsurance is not capital efficient.
The forecast summary excerpts for the two segments are illustrated by accident year; CVs are highlighted. The CV of the Net of Reinsurance is the same as than the Gross in total, but higher by individual accident years. The latter is a result of the higher process volatility in the Net of Reinsurance data.

The risk capital as a percentage of the mean is almost identical for Gross and Net of Reinsurance – there is no advantage in this outward reinsurance program in terms of capital efficiency for the cedant.

3.3. Key learnings
- Not all reinsurance programs are capital efficient for the cedant – two contrasting examples were given;
- ICRFS-Plus™ modelling frameworks can be used to assess existing outward or inward; reinsurance, retrospective or prospectively
- Reinsurance structures can be examined from the point of view of the cedant or the reinsurer.
4. Layers

4.1. 1M, 1M xs 1M, 2M

In this example, data are split into three layers - paid losses with each individual paid loss limited to 1M, paid losses with individual losses between 1M and 2M (1M xs 1M), and paid losses with individual losses limited to 2M. All losses are capped at 2M.

We observe:

- Calendar year trends in the layers are unique. The calendar year trend in the layer 1M xs 1M are not statistically different from zero whereas the layers Lim 1M and Lim 2M show statistically significant positive trends;
- There is no gain in risk capital efficiency ceding the layer 1M xs 1M retrospectively;
- There is a small gain in risk capital efficiency when ceding the layer 1M xs 1M prospectively;
- If both reserve and future underwriting years are considered, then there is no gain in risk capital efficiency in ceding 1M xs 1M both retrospectively and prospectively.

The corresponding MPTF model displays are shown below; these models include constraints in the accident direction.

The key features of interest are that in the claims limited to 1M and in the claims limited to 2M, there is evidence of a calendar year trend, however in the losses 1M xs 1M there is no statistically significant calendar year effect.

The Layer 1M has a higher inflation rate than 2M, and 1Mxs1M has an inflation rate that is statistically insignificant. If the only available array is 1Mxs1M then it would be prudent not to set the inflation to zero, as process volatility is high.

The calendar year trend is estimated in the model, even though it is not statistically significant, so correlation is maintained between the calendar year parameters. The relationship between the parameters is important since the two pieces are highly correlated and the calendar year trend is significant in 2M.

The other two directions, development year and accident year, are similar although there is a positive development trend in 1M xs 1M where the other layers have a zero trend.

The layers are highly correlated and the model above uses the high correlation between the layer limited to 1M and 1M xs 1M. Including the correlation between 2M and the other two pieces would be inappropriate since this would be equivalent to using repeated measurements; all the information to estimate 2M from 1M + 1M xs 1M is already in the model.
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The efficiency of the layers can be readily evaluated by comparing the CVs (highlighted) of the forecasts for the layers along with the percentage of risk capital over the mean for each layer.

From the table below for the reserve forecast, the respective CVs are essentially the same for all layers. This indicates there is no gain in efficiency to cede the 1M xs 1M layer.

The above confirms the hypothesis stated above. The level of risk capital required, as a percentage of the total capital, is the same for all layers. For this to be true, the V@Rs must be approximately additive for the two layers (1M, 1Mxs1M) compared with 2M - illustrated below. The average percentage difference in the V@Rs (1M + 1Mxs1M versus 2M) is only 2.3%.

This result does not hold for pricing the next underwriting year; instead there is an efficiency gain, albeit slight, from ceding the 1M xs 1M layer. This arises due to the high process volatility associated with the first three development periods particularly.

The forecast tables for the next underwriting period (1999) are shown for the three layers below.
The risk capital efficiency at 95% for the future underwriting year is illustrated below. There is 1.25% gain in efficiency from ceding 1M xs 1M. This difference, although small, is significant. If reserve and underwriting risk are considered then no benefit is found since the reserve component dominates.

Risk capital requirements based on Value-at-Risk (V@R) are shown below for the three layers for the future underwriting year 1999 for various percentiles. Here the average difference between the V@Rs (1M + 1Mxs1M versus 2M) is 5.5%; a significantly higher figure.

Forecast tables for the next three underwriting periods (1999–2001) are shown below for the aggregate of 1M and 1M xs 1M. The distribution means and standard deviations are for the sum of correlated log normals. The individual layers can also be viewed enabling pricing to be conducted on either the individual layers or the aggregate of the layers. This pricing can also be compared with projecting 2M on its own.
4.2 CSLE-02: evaluating optimal layer retention and pricing layers
The data for this case study comprises a set of layers and the ground up data.

Layers to be evaluated comprise:
- Limited to 100k; 100k+
- Limited to 10k; 90k XS 10k; 100k+
- Limited to 50k; 50k XS 50k; 100k+

We show:
- Not all layer combinations are efficient;
- Trends are different in each layer; it is important to consider the composition of the trends being ceded.
- Correlations between the different layers must be considered.

4.2.1. Model structure
Models are designed for the ground up data and each layer and a single composite model identified in the MPTF modelling framework. Note that the layers comprising each segmentation of the ground up data retain their process correlation, but there is no correlation set between different layer compositions. In order to facilitate correlations between the segments and the layer 100k+ multiple copies of the 100k+ data are included in the composite model. The models for the 100k+ data vary slightly according to the correlations between them and the other layers; there are no material differences in the projections for the 100k+ layer.
4.2.2. Process (volatility) correlations

Below are the process volatility correlations between the segments. Note each layer group are in independent clusters and multiple copies of the 100K+ layer are included to correctly account for correlation between the different data splits.

From the above models, we can then assess the capital efficiency of ceding/retaining different layers from the perspective of the insurer and reinsurer. Inward reinsurance can then be priced accordingly, or outward reinsurance purchased, with full knowledge of the benefits.

Note also, the benefit of identifying the trends applicable to the different layers. The most optimal split has a capital efficiency of the aggregate (sum of layers) at 7.19%. This is lower than the model for the ground up where the data are modelled as one paid loss array. There are different trends identifiable in each layer and the additional information available in the split aids modelling.
4.2.3. Risk capital efficiency of each layer and the ground up

The following table shows the risk capital efficiency of each analysis.

<table>
<thead>
<tr>
<th>Segment</th>
<th>Mean</th>
<th>V@R (99.5%)</th>
<th>Risk Capital % of Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground-Up</td>
<td>442,281</td>
<td>36,953</td>
<td>8.36</td>
</tr>
<tr>
<td>0°-100</td>
<td>371,663</td>
<td>23,165</td>
<td>6.23</td>
</tr>
<tr>
<td>100k+</td>
<td>53,253</td>
<td>14,211</td>
<td>26.69</td>
</tr>
<tr>
<td>Sum (0°-100 + 100k+)</td>
<td>424,916</td>
<td>30,562</td>
<td>7.19</td>
</tr>
<tr>
<td>0°-10k</td>
<td>168,273</td>
<td>16,031</td>
<td>9.53</td>
</tr>
<tr>
<td>90k xs 10k</td>
<td>206,013</td>
<td>14,884</td>
<td>7.22</td>
</tr>
<tr>
<td>100k+</td>
<td>55,891</td>
<td>14,593</td>
<td>26.11</td>
</tr>
<tr>
<td>Sum (0°-10 + 90k xs 10k + 100k+)</td>
<td>430,177</td>
<td>31,082</td>
<td>7.23</td>
</tr>
<tr>
<td>0°-50k</td>
<td>338,358</td>
<td>26,431</td>
<td>7.81</td>
</tr>
<tr>
<td>50k xs 50k</td>
<td>49,595</td>
<td>6,279</td>
<td>12.66</td>
</tr>
<tr>
<td>100k+</td>
<td>55,304</td>
<td>14,530</td>
<td>26.27</td>
</tr>
<tr>
<td>Sum (0°-50 + 50k xs 50k + 100k+)</td>
<td>443,257</td>
<td>35,491</td>
<td>8.01</td>
</tr>
</tbody>
</table>

The most capital efficient split for the cedant (marked in bold) out of the layers considered above is to retain all claims less than 100k and cede the excess. This split also happens to be the best in terms of modelling efficiency; the least efficient is to model the ground up data as one piece without segmentation into layers.

The most capital efficient layer for the reinsurer is the 90k xs 10k layer.

4.3. Key learnings

- Splitting the data into layers can yield surprising results; not all layer combinations are efficient nor is it necessarily efficient to split the data into layers at all;
- Trends are generally expected to be different in each layer, but care must be taken with understanding the resultant trends in the models. It is important to know what risks are being ceded.

The above information is essential when either assessing or pricing reinsurance. Using ICRFS-Plus™ many layer combinations can be analysed in the MPTF modelling framework.
5. High Severity / Low Frequency

5.1 Example: Pricing 5M xs 5M from data limited to 5M and data limited to 10M

Pricing high severity / low frequency data is typically problematic. In this example, the 5M xs 5M layer is replete with zeroes and pricing this layer on its own would be guesswork. The strategies encompassed in this section show how sparse upper layers can be projected utilising information in the ground-up or other limited data. We use the information in the limited 5M and limited 10M data to calculate the total reserve and to price the next underwriting year.

Consider the task of pricing 5M xs 5M from the data:

If only this data were available, then the task would be incredibly difficult if not impossible. It would be inappropriate to fit a single distribution to the data as the distributions in each cell are different. Without knowledge of how the cells are related, pricing becomes reduced to guess work.

The data are sparse, scattered, and when the limit is breached: extreme.
In addition to the 5M xs 5M layer, we have data at four different layers: Limited to 1M, Limited to 2M, Limited to 5M, and Limited to 10M. Since the model structure was found to be the same in all layers (parameters are located at the same point), correlations between the layers can be used with no double counting effect (unlike other examples presented in this document) due to the working of the estimation algorithms.

The process correlations between the layers were used, and constraints between parameters were applied where appropriate.
Pricing

The model displays corresponding to each layer limit are shown below.

![Model Display](image-url)
The model displays show common parameters in the development direction between all layers (red bars). The calendar parameters vary slightly in the upper two layers (statistically they are the same) and the process volatility is also unique to each layer. In all cases, parameter changes (including heteroscedasticity) occur at the same place.

We forecast distributions for 5M xs 5M by subtracting Limited 5M from Limited 10M. The forecast table is shown below.

A key feature is that the mean for each cell is low (most entries are zero), but the standard deviations for each cell are extremely high – thus allowing for the extreme cases observed in the 5M xs 5M data triangle.
As with any other ICRFS-Plus™ Probabilistic Trend Family model, we can price future underwriting periods. Again we use the models for the data limited to 10M and the data limited to 5M to produce the forecast for the difference (shown below).

The outstanding distribution correlation is negative due to the subtraction of the limits. The correlation in the reserve distributions is very high (as expected).
The density plot of the differences between the layers is depicted above. The negatives emerge as a result of the differences (negatives can occur in the XS layer – see original data) but are unlikely to happen. If the layer to be priced was outside the interval of the data, then the best way to approximate this layer would be to predict the mean for the layer using the trends in the projections from lower limits. The means (and other distribution characteristics) are then modelled and subsequently projected into the high limits.

5.2. Key learnings

- If the ground up data are available, then high severity / low frequency layers can readily be priced using ICRFS-Plus™;
- If losses being priced are outside the range of the excess data, then approximations of the excess distribution (mean and standard error) can be approximated via projection of lower layers.
- Fitting a single severity (claim size) distribution to all individual losses is inappropriate since the severity distributions vary by development, accident, and calendar period. That is, the frequency/severity distributions are not homogenous across periods.
6. Adverse Development Cover

6.1. LOB 1 through 6

This case study illustrates how Adverse Development Cover (ADC) can be priced or evaluated for capital efficiency for a single LOB or multiple LOBs. Complex reinsurance structures are discussed including, varying limits on individual LOBs as well as the aggregate of LOBs and multiple limits by accident year. Retrospective and prospective reinsurance structures can be considered.

The MPTF modelling framework projects correlated log-normal distributions by cell for each LOB. The aggregate forecast table contains means and standard deviations by cell as shown below. The distributions of the aggregate are obtained by simulating from the correlated log-normal distributions to obtain distributions by cell, period total, and total reserve.

LOB 4 represents 7.88% of the total mean reserve but will require a significant proportion of the risk capital as this line has the largest CV and second largest reserve standard deviation.
Risk capital is particularly important for the next three calendar years, thereafter it is less important. However, total allocation of risk capital based on the variance-covariance formula allocates significant capital to LOB 4—the main driver of risk in the later calendar years.

Capital efficiency of the aggregate of the six LOBs is 9.43%. Most of the risk capital is allocated to LOB 4; LOBs 2 and 6 receive little risk capital.

Consider Adverse Development Cover for the aggregate of the six LOBs. Suppose the lower attachment point (M1) is the mean of the total reserve loss distribution and the upper attachment point (M2) corresponds to the 90th percentile of the total aggregate loss distribution. That is: $M_1 = 660M$ and $M_2 = 706M$ respectively. Sample statistics for the ground up, insurer, and outward reinsurer follow.
Based on limits described above, the CV of the insurer shows a slight improvement (from 5% to 4%). What about capital efficiency? These results are provided below, where see that the insurer, assuming this ADC policy, now requires a risk capital percentage of 4.23% of the mean, while the outward reinsurer requires 283% of the expected mean.

<table>
<thead>
<tr>
<th></th>
<th>Insurer</th>
<th>Reinsurer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample Mean</td>
<td>674.47</td>
<td>12.38</td>
</tr>
<tr>
<td>V@R 95%</td>
<td>27.42</td>
<td>34.32</td>
</tr>
<tr>
<td>% Risk Capital</td>
<td>4.23</td>
<td>283.35</td>
</tr>
</tbody>
</table>

The insurer does show considerable improvement in terms of the % risk capital of the mean versus the % risk capital of the mean for the aggregate of the six LOBs (9.43% from the risk capital allocation graph presented earlier).

If the outward reinsurance is accessed (47% probability based on simulations) there is a good chance (26%) that all the reinsurance capital will be required.

6.2. LOB 4

The most volatile line of the six LOBs in the previous section is LOB 4. There are two reasons for this. Firstly, the calendar year trends are very volatile - the most recent trend (14% + 3.51%) has a CV of 25%! The uncertainty associated with the trend has significant impact on the standard deviations and associated risk capital. The second reason is that the process volatility is relatively high – especially in the late development years.

The model display highlights both these features.

The standard deviations of the calendar year totals increase significantly by the calendar periods. This is the reason that a high level of risk capital is allocated to LOB 4 when considering the six lines jointly – the parameter uncertainty has major impact. Since the uncertain trend is in the calendar year direction, the most recent accident years are the most adversely affected (see below).
The company obtains significant risk diversification credit by writing this line in combination with the other five lines. The total percentage risk capital is 9.43% versus 68% for LOB 4 on its own!

Is there any benefit to obtaining reinsurance for LOB 4 on its own and how do different retention levels affect the efficiency of the risk capital required? The following table shows the benefit to the insurer for various retention levels compared to the ground up data at the 95% risk capital level. The insurer is assumed to take all losses up to the projected mean loss (the respective M1). The outward reinsurer takes any losses between the mean and various attachment levels (the respective M2). Any losses above the upper attachment point are also taken by the insurer. The benefit in terms of capital efficiency is shown below – in particular, the % risk capital required by the insurer after taking outward reinsurance. Note for the final entry, M2 is above the 95% quantile for the total losses (94.5M), thus significant benefit is obtained by the insurer compared to the other levels.

This information is also critical to the outward reinsurer; any pricing for this LOB on its own should be priced conservatively due to the level of risk taken.

The diversification credit obtained by writing LOB 4 jointly with the other LOBs (then purchasing outward reinsurance) can be contrasted with purchasing outward reinsurance for LOB 4 on its own. Subsequent adverse development cover can then be considered on the new aggregate of the six lines where LOB 4 is now Net of the Reinsurance. The highest attachment point for M2 (above) was used for LOB 4 for comparison purposes.
The above corresponds to the distribution of the aggregate of the six LOBs (no outward reinsurance). The V@R at 95% is 62.2M (using the Kernel – a smoothed form of the sample), and this corresponds to the risk capital efficiency percentage of 9.43%.

The two aggregate loss distributions with and without outward reinsurance on LOB 4 are overlaid below (blue corresponds to the distribution of the total losses without reinsurance, the light grey to the distribution of the total losses with outward reinsurance on LOB 4: $M_2 = 97.3M$ as shown previously).

The difference in the quantiles (15.3M) is split between the difference in means (6.9M) and the difference in V@R (about 8.4M). The outward reinsurance on LOB 4 translates into a gain in capital efficiency for the aggregate; the risk capital as a percentage of the mean drops from 9.43% to 8.24%.

Similarly, the CV of the aggregate distribution is now 4.9%; down from 5.5% shown previously.

The above results also give a value of the outward reinsurance in respect of the aggregate loss distribution. The insurer is expected to compensate the outward reinsurer to a value above the mean losses for the outward reinsurer – however the total compensation (mean + risk) should not exceed the difference in quantiles at the level of risk the insurer is evaluating (here 95%).

This outward reinsurance structure may be more cost effective than writing adverse development cover against the aggregate of the six lines.
6.3. Pricing adverse development cover across differing limits
Consider the task of pricing adverse development cover where the following layers are applied. The same data as used for the 5M xs 5M analysis previously is considered – see previous model displays.

Reinsurance is required on the aggregate of the layers where they apply. This can also readily be done in the MPTF modelling framework. The aggregate forecast of the four layers is shown below.

The forecasts for each layer correspond to the accident years they apply for. For instance, Lim 5M applies to 1994–1996 and 2000–2001. These cells are highlighted below and match the aggregate figures above for those same periods.
Pricing

The complete forecast distributions can be analysed by individual accident years and in total – just like any other forecast!

The total reserve distribution is highly skewed; skewness which is primarily driven by the volatility in the Limited 5M layer – especially AY 2001. From the model displays, shown previously, the 2001 year was unusually high - an unfortunate time to have a high limit. Reinsurance at a lower threshold was subsequently taken out post-2001.

Risk capital is primarily allocated to the Lim 1M layer (most years) followed by the Lim 5M layer. Any combination of layers can be considered for comparison purposes – similarly construction of multiple excess layers can be considered from the perspective of an inward reinsurer.

6.4. Key learnings
• Adverse development cover can be evaluated against a single LOB or a multitude of LOBs;
• Complex reinsurance structures can be analysed where multiple layers apply for different underwriting years;
• Subsequent updates of reinsurance limits can be evaluated – namely is there any benefit to increasing / decreasing the level of reinsurance for the cedant.
Changing the face of actuarial solutions forever

7. Adverse development cover as a means of protection against conservative scenarios

In this case study, we consider two scenarios - a reasonable scenario and a conservative scenario where the conservative scenario adds additional prudence for adverse inflationary trends. Reinsurance can be considered as an option to provide adverse development cover against the adverse trends arising. We illustrate this concept in the context of LOB 1.

The calendar year trends change several times in the last ten years. A number of scenarios may be considered for the future. The actuaries most reasonable scenario is to continue with the 6.91%±2.15% calendar year trend until run-off is complete. However, based on their knowledge of the business, there is a small chance (5%) of the 35.3%±4.2% calendar year trend (previously observed 2000~2002) emerging in the next year, zero the year after, then returning to the 6.91%±2.15% calendar year trend for the remaining run-off period.

There is a substantial difference in the forecasts arising from these two scenarios. The reasonable scenario estimates the mean of the total loss reserve distribution of 87.8M whereas the conservative scenario places the mean of the total loss reserve distribution at 113.5M. This conservative mean estimate is around 29% higher than the reasonable estimate. The magnitude of the difference is such that it would be unwise to reserve the full conservative mean as there is only a 5% probability of this level of capital being required and the actuarial team states the 87.8M estimate as the best estimate.

An appropriate option under these circumstances is to use the conservative scenario as a basis for purchasing retrospective outward reinsurance. The protection in this scenario is against an adverse calendar year trend arising. Therefore, the most affected years are the recent years. There is likely little advantage in adding retrospective cover to the earliest years.

The insurer has sufficient risk appetite to retain losses up to the 75th percentile of the total loss reserve distribution of the reasonable scenario. What level of adverse development cover is required to maintain the same risk capital appetite against the conservative scenario also at the 75th percentile? What is the most efficient way of realising this cover?
There are at least two options. The first option is to set the attachment points so that the insurer’s risk capital is risked first, the second option is that the reinsurance applies first then the insurer’s risk capital applies. The second option is what we consider here, although the reinsurance is probably more costly for this option since it applies sooner.

First of all, we observe that the conservative scenario emerging would potentially be catastrophic for the insurer if the reasonable scenario was reserved without any further reinsurance program. The mean total reserve from the conservative scenario is sitting at the 97.4% quantile for the reasonable scenario.

The liability streams for the two scenarios, reasonable and conservative respectively, are as follows:

It is immediately apparent that the critical differences are in the first few calendar years. This is where the company pays the most money and this is where the high calendar year trend is emerging in the conservative scenario.

An infinite number of possible reinsurance structures could be created to address this scenario. The focus, however, should be on where the primary risks of loss are.

The principle accident years affected by the adverse calendar year trend are the most recent two accident years. Therefore, the outward reinsurance structure likely most optimal from the cedant’s perspective is to price reinsurance for only the most recent two accident years (and the next underwriting year as the line is still written).
The leading authority in long tail liability risk management

The differences in the mean projections between the reasonable and conservative scenarios is 46.7M; roughly an additional 30% on the mean estimate from the reasonable scenario.

The capital the insurer is prepared to risk for adverse trend development is 13.1M - the losses up to the 75th percentile of the reasonable scenario. The outward adverse development cover for protection against the conservative scenario (severe adverse trends), should be designed so that the V@R at the 75th percentile for the conservative scenario is also 13.1M. Using the strategy where the reinsurance capital is accessed before the insurer’s capital, the attachment points used are the reasonable scenario mean for the lower limit (162.7M) and the upper limit given by the 75th percentile less 13.1M (using the Kernel density below: 227.3M - 13.1M = 214.2M).

The outward reinsurance table from the reasonable scenario with these attachment points is as follows. This is the most likely scenario.
The outward reinsurance has two components, it lowers the exposure of the insurer to adverse development arising from process volatility assuming the reasonable scenario emerges and, additionally, is highly effective for protection against the adverse inflationary trends described by the conservative scenario.

The effectiveness is seen immediately in the comparative outward reinsurance analysis from the conservative scenario (5% likelihood of arising).

For the same attachment points, the summary statistics are shown above. The expected losses of the outward reinsurer are substantially higher (the 8M increases to 38M). Similarly, the expected losses of the insurer are substantially higher than the best estimate from the reasonable scenario, but the difference (171.5 - 162.7 = 8.8M) is within the 13M capital the insurer was prepared to allocate to cover adverse development arising from either adverse inflationary trends or process volatility.
7.1. Key learnings

- Adverse development cover can be purchased as means of protection against adverse trends arising from the reasonable forecast assumption;
- Complex reinsurance structures can be analysed when considering reinsurance against adverse inflationary trends emerging.
- Inward reinsurers can evaluate the effect of different economic scenarios against the cedant’s forecast assumptions;
- A fair price can be negotiated when both parties have access to the critical information.