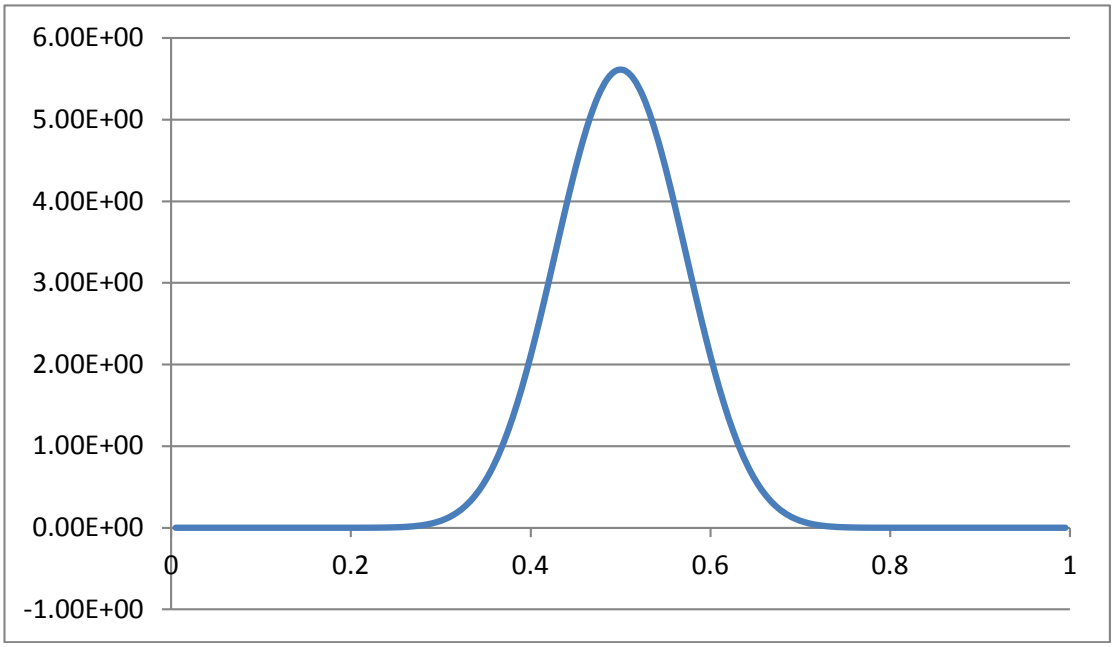
Look-Ahead Stats

Experiment 1:

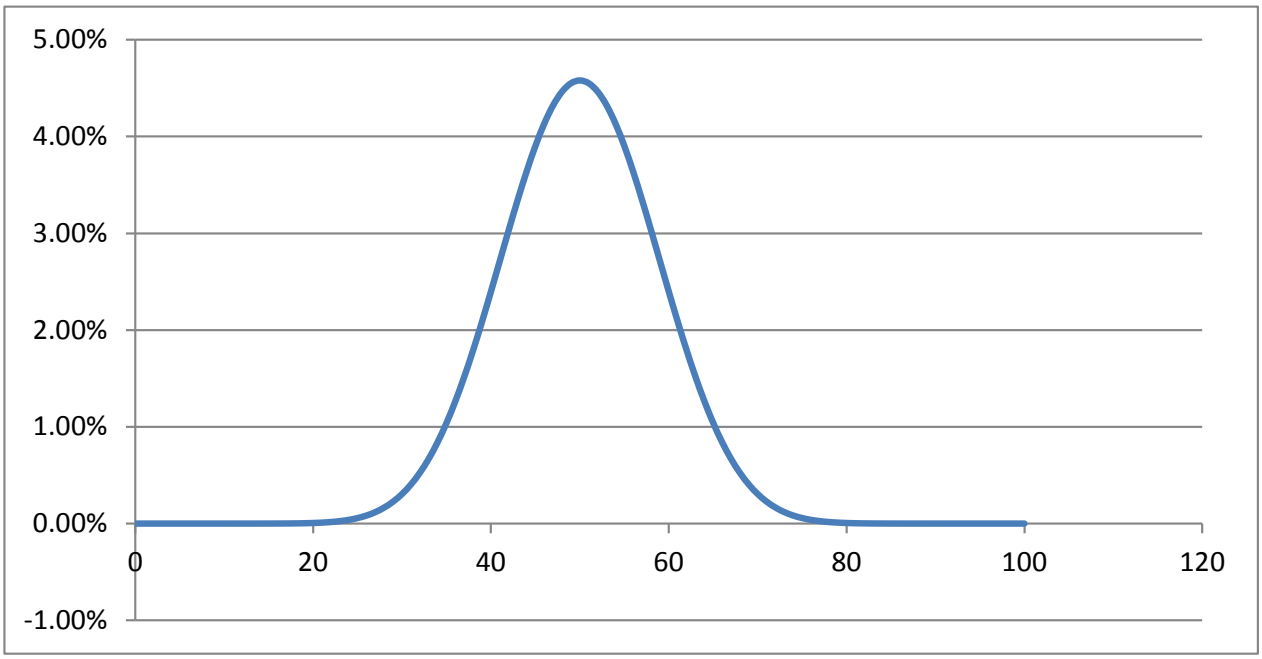
Take a "coin" and flip it 100 times.

Context: You don't know much about this "coin" beforehand, it might be an electronic coin, it might be a biased physical coin, but you do know that the outcome of every "toss" is independent of the previous toss (it resets each time) and that the probability is fixed.

Your uncertainty can therefore be expressed in terms of a "prior" distribution for the probability that when tossed it yields heads.

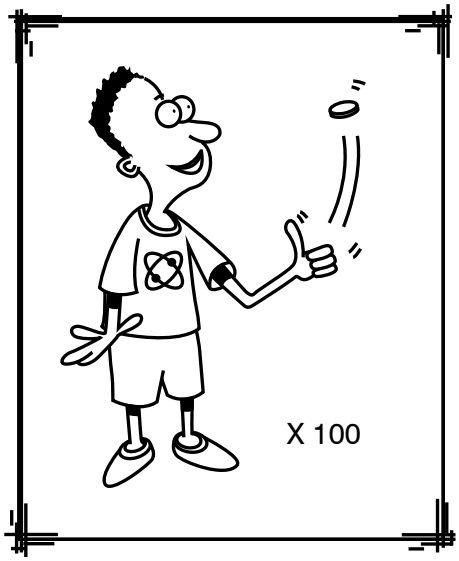


Here is the prior distribution for the probability of heads.
(Beta(25,25); mean = 0.5 standard deviation = 0.07)
From this you can compute the probability distribution for the outcome of the experiment, measured as the number of Heads.



Here is the distribution for the outcomes; mean = 50.0, standard deviation = 8.57

What these graphs show is the knowledge that You-Now have about the projected outcome of these experiments:



You, thinking about the experiment, which consists of 100 coin tossings.

Experiment 2 (same experiment):

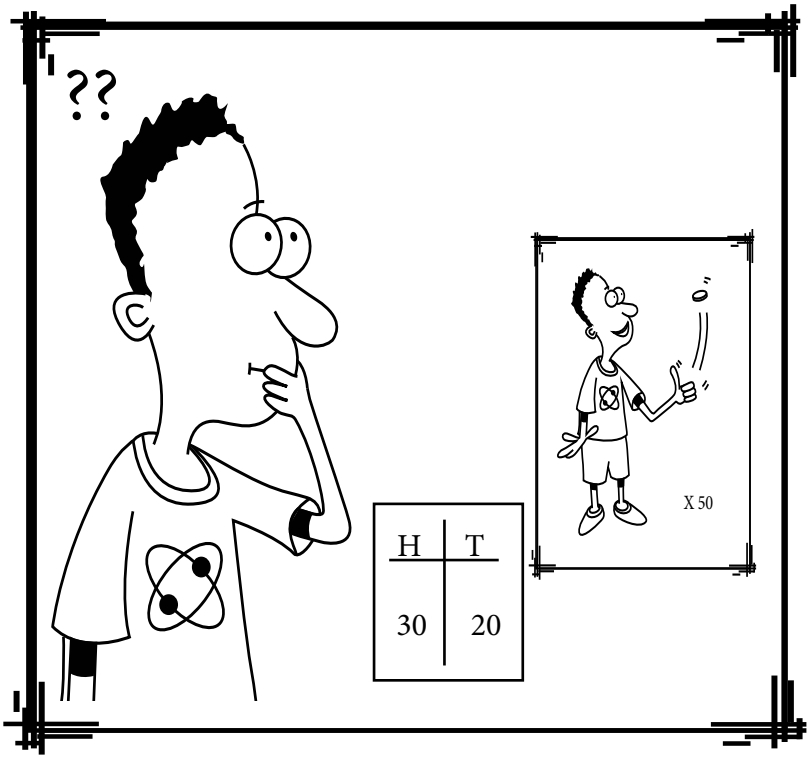
Now let's view exactly the same experiment in a different way.
This time you decide that after 50 "coin" tosses you will stop and take stock and create new predictions.
The time is still Now, the only difference is that you now have something new to think about. The new thing is the future-You that will be around after 50 coin tosses have been completed.

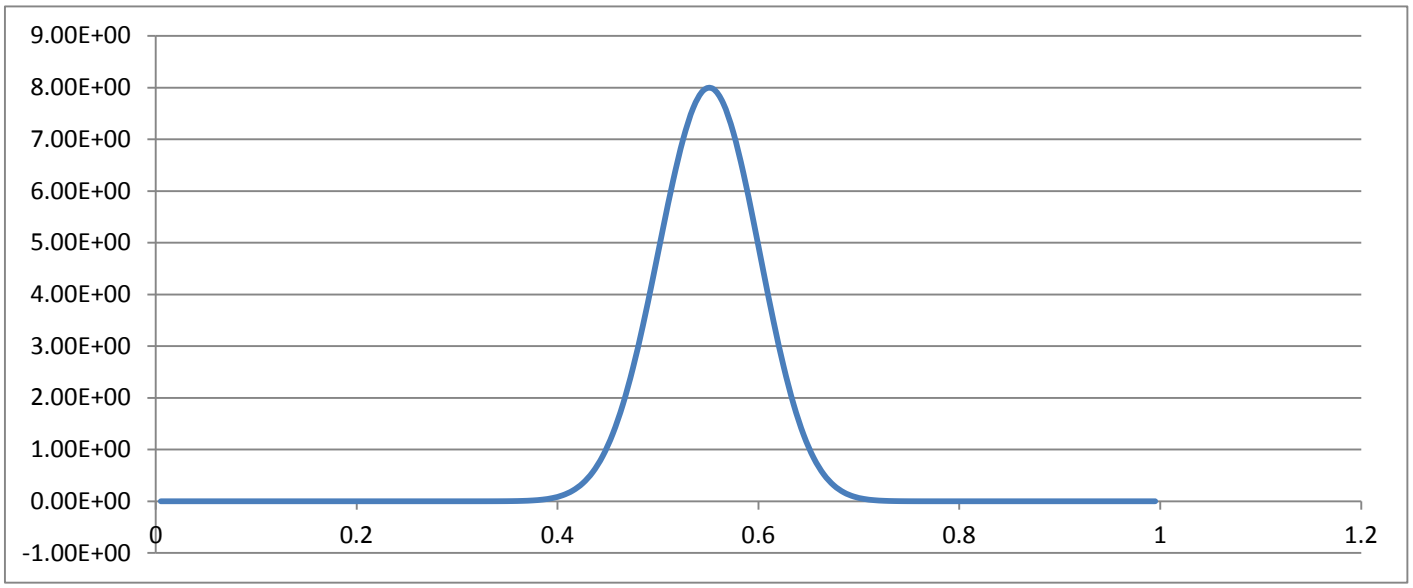
There are in effect now 51 different future "you"s (in 51 different parallel universes, if you like) who will have 51 different updated estimates of the coin probability.
Let's consider one of these.

The future-You that sees the outcome of the first fifty tosses producing 30 Heads. This one has reason to re-estimate the likely probability of heads as higher than 0.5.

You thinking about the future-You that has recorded 30 Heads after 50 coin tosses.

In fact his "posterior" distribution looks like this:

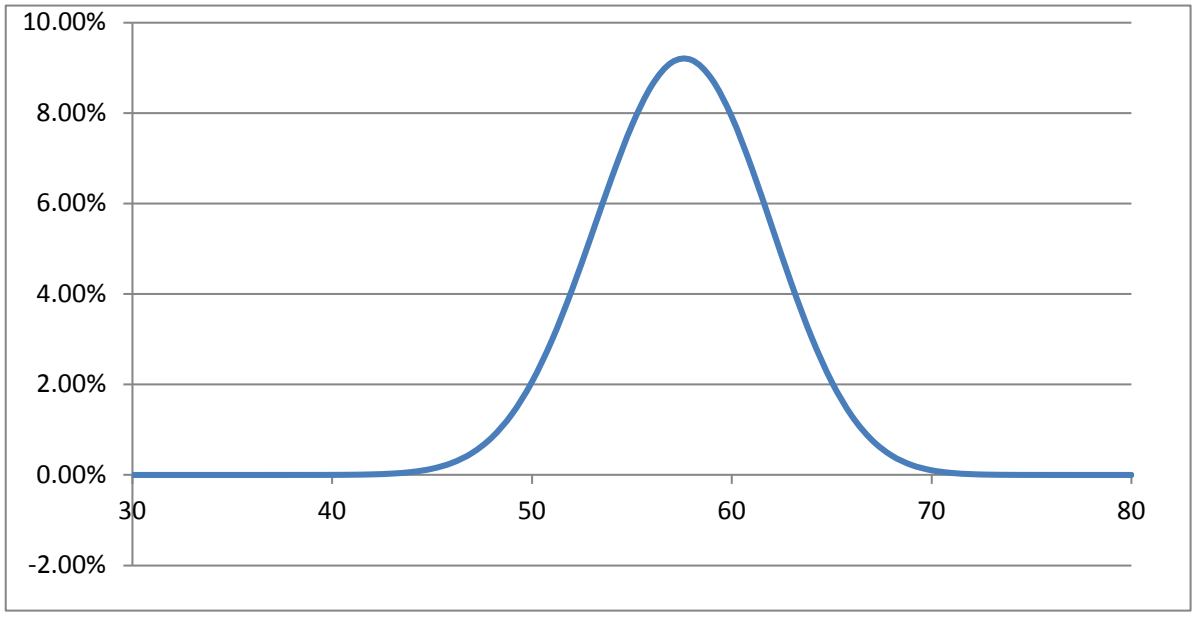




Probability distribution for the probability of heads, after 50 tosses have yielded 30 heads. (Beta(55, 45); Mean = 0.55, standard deviation = 0.05)

The uncertainty as measured by standard deviation has dropped by 28%, from 0.07 to 0.05, due to the new information that has been gained. But this new information is only possessed by the future-You, the Now-you doesn't have it.

The predicted final distribution of Heads made by this particular future-You is this:

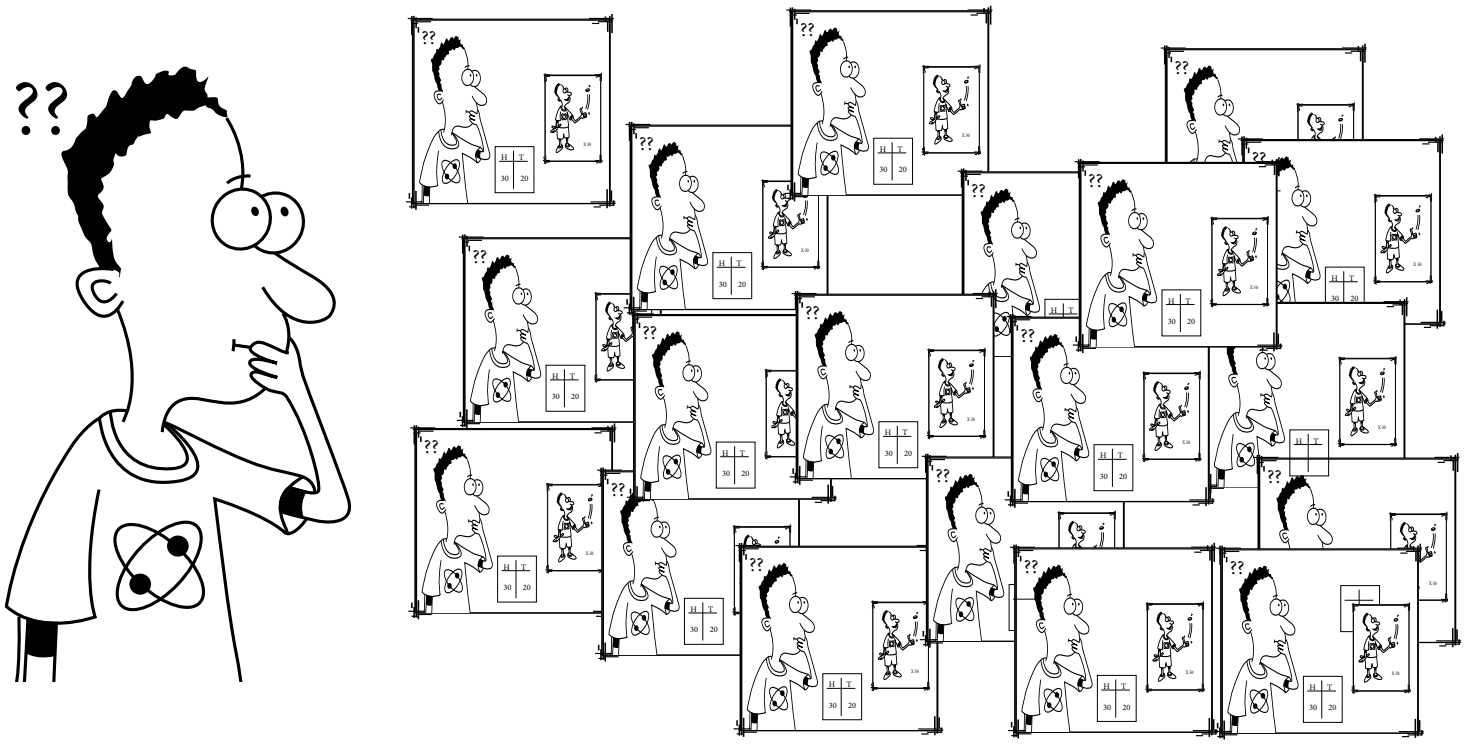


Mean = 57.5 Standard deviation = 4.29

The uncertainty for this future You is as measured by St.Dev. is about half of what it was before.

Two factors make for this reduction:
(i) the coin probability is now better known i.e. reduction in parameter uncertainty and
(ii) there are only 50 unknown events to predict instead of 100, hence a reduction in process variance.

But this is just one future-You, we need to consider all of them.



What you know is that if you take the mean of all the means that will just be 50, the number you first had, because no new information has been added by way of this thought experiment.

You also know that while, each of these 51 future-Yous will have a different value for the future standard deviation these values will all be lower than your initial Std. Dev. of 8.57 because factors (i) and (ii) listed above are still true in every case.

If you work it out, the average St. Dev. among all the 51 is: 7.00 [This is actually the square root of the average variance.]

This is 18% less than the original. Where has the lost uncertainty gone?

Answer: It is hidden in the standard deviation of the outcomes of the first 50 tosses.

All we have really done is split the total amount of variation into two components, (1) that belonging to the first 50 tosses (St.Dev. = 4.95) and (2) the "50-toss-look-ahead" average St.Dev. which is 7.0